Automatic Generation of Serial Robot Architectures from Required Motion Directions

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ABSTRACT

This paper presents an approach for the automatic generation of serial robot architectures, which, in comparison to existing methods, is easier to implement in a computer aided design process. From the required Cartesian translational and rotational motions along the axes of the robot base reference frame, the feasible serial robot architectures are generated using a discret set of Denavit-Hartenberg parameters and the rank of the Jacobian matrix. The method begins with the definition of the required motion through a required motion vector, which can be manipulated to take into account possible orientations of the robot base. The complete algorithm as well as an application example are given. Finally, the number of solutions generated with the proposed method are listed for serial manipulators from two to five degrees of freedom.

1 INTRODUCTION

One main issue to be considered within a robot design process is to find an appropiate architecture that accomplishes desired degrees of freedom (DOF). The structural synthesis of robotic manipulators, sometimes called type synthesis, topological synthesis, or conceptual design of the architecture, aims to find the number, type, and orientation of the robot joints in order to generate a manipulator with required DOF at the end effector (EE) [25].

Among the current methods to carry out the structural synthesis of a manipulator, the most widely used are the graph theory [23, 12, 2], position and orientation characterization (POC) [29, 30, 28], screw theory [14, 17, 18, 16, 11, 15, 20, 19, 5], groups of displacement [13, 1, 22], and linear transformations [10, 9, 8]. Most of these approaches have been applied in the design of parallel manipulators but can be also applied to serial manipulators. One of the first approaches specific for serial robots is shown in [27] where possible configurations of 6DOF decoupled serial manipulators are suggested using revolute (R) and prismatic (P) joints . There, the synthesis is separated into two parts: the first takes into account the three translational DOF, the second part considers the three orientational DOF given by a wrist located at the tip of the first part of the mechanism.

More recently, the rule-based design of serial kinematic chains has been proposed, as in [3], where serial configurations for Schoenflies motion generators are given. A couple of rules for assembling a manipulator up to 6DOF are deduced as well in [21] using the screw theory and comparing the resulting twist with the motion constraints imposed. In [24] the Clifford algebra exponential design equations of a chosen topology are equated to a set of task positions and solved to find the length of the links and the orientation of the joint axes. Additionally, in [6] six 6DOF serial robotic manipulator families with only revolute joints are proposed applying travelling coordinate systems (TCS) to describe the geometry of the manipulators and evolutionary morphology (EM) [7] to generate them.

Except EM, the existing approaches are difficult to include in a robot computer aided design process fulfilling a complete set of requirements such as workspace, accuracy or dynamic. Therefore, it is necessary to develop an approach for the automatic structural synthesis of serial robot manipulators allowing for determining the parameters, through which the obtained architectures can be compared. In this way, the first step is the automatic generation of all suitable structures that will be evaluated in further steps to complete the structural synthesis. EM allows for creating solutions to the structural synthesis problem in a systematic way [7]. However, the relative orientation between adjacent joint axes can be described only as parallel or perpendicular and it does not allow for taking into account other possible relative orientations as in [24]. Furthermore, EM does not allow for considering the desired direction of the required motion but only the number of rotational and translational DOF. Hence, an approach for the automatic generation of serial robot architectures from required motion directions is proposed based on the desired direction of each DOF. The method is based on EM but taking into account the direction of the desired DOF. Instead of morphological operators, it finds all suitable architectures from a discrete set of Denavit-Hartenberg (DH) parameters. The new algorithm is also able to handle the DH parameter α to describe the relative orientations between two adjacent joint axes as an alternative to the condition of parallel/perpendicular used by the current methods. The paper is organized as follows. Firstly, the calculation of the Jacobian matrix and the forward kinematics of serial robots is presented. Secondly, the definition of a suitable architecture for being solution of the structural synthesis is introduced. Then, the algorithm to generate suitable solutions for this problem is shown together with an application example. Finally, the results of the algorithm for required motions with up to 5DOF are tabulated and commented.

2 KINEMATICS OF SERIAL MANIPULATORS

The velocity of the EE of a serial manipulator can be described using the origin $\mathbf{O}_n$ of its reference frame $(\mathrm{RF})_n$ as a reference point [26]:

$$\boldsymbol{\xi} = \begin{pmatrix} \mathbf{v}_n \\ \boldsymbol{\omega}_n \end{pmatrix} = \sum_{i=1}^{n} \left(\dot{\theta}_i \begin{pmatrix} \mathbf{z}_{i-1} \times (\mathbf{O}_n - \mathbf{O}_{i-1}) \\ \mathbf{z}_{i-1} \end{pmatrix} + \dot{d}_i \begin{pmatrix} \mathbf{z}_{i-1} \\ 0 \end{pmatrix} \right), \quad (1)$$

where $\mathbf{v}_n$ and $\boldsymbol{\omega}_n$ are the linear and angular velocity of the EE, $\dot{\theta}_i$ and $\dot{d}_i$ correspond to the joint velocity of the i-th joint, $\mathbf{z}_{i-1}$ represents the axis of the i-th joint and $(\mathbf{O}_n - \mathbf{O}_{i-1})$ is a position vector from the origin $\mathbf{O}_{i-1}$ of the reference frame of the $(i-1)$-th link to the point $\mathbf{O}_n$ as can be seen in Fig. 1. Equation (1) corresponds to the differential kinematics of a serial manipulator and can be also written as:

$$\boldsymbol{\xi} = \mathbf{J}\dot{\mathbf{q}} \quad (2)$$

with $\mathbf{J} = (\mathbf{J}_1, \mathbf{J}_2, \ldots, \mathbf{J}_n)$ being the Jacobian of the manipulator and $\dot{\mathbf{q}} = (\dot{q}_1, \dot{q}_2, \ldots, \dot{q}_n)^{\mathrm{T}}$ being the joint rates, which correspond to $\dot{\theta}_i$ in case of a revolute joint and $\dot{d}_i$ in case of a prismatic joint. $\mathbf{J}_i$ for $i = 1,2,\ldots,n$ is the i-th column of the Jacobian and represents the effect of the i-th joint rate on the velocity of the EE.

The position $\mathbf{O}_{i-1}$ and orientation $\mathbf{z}_{i-1}$ of the i-th joint as well as the EE reference frame $\mathbf{O}_n$ can be calculated using the forward kinematics of the manipulator:

$$\begin{pmatrix} \mathbf{z}_{i-1} \\ 0 \end{pmatrix} = {}^{0}\mathbf{T}_{i-1} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} \mathbf{O}_{i-1} \\ 0 \end{pmatrix} = {}^{0}\mathbf{T}_{i-1} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}. \quad (3)$$

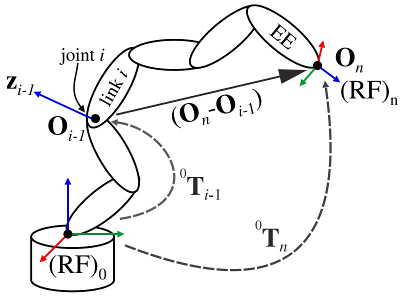

Fig. 1 Vectors to describe the location of the joint axes

The matrix ${}^{0}\mathbf{T}_{i-1}$ is the homogeneous transformation matrix between the reference frame of the base $(\mathrm{RF})_0$ and the reference frame of the $(i-1)$-th link $(\mathrm{RF})_{i-1}$, it can be calculated using the DH parameters as shown in [4].

3 DEFINITION OF SUITABLE ARCHITECTURES

The required degrees of freedom are defined through the required motion vector $\boldsymbol{\xi}_{\mathrm{req}}$, that corresponds to the column vector $\boldsymbol{\xi} \in \mathbb{R}^6$ in (1) in which some of its terms have to be zero for any $\dot{\mathbf{q}}$, i.e. $\boldsymbol{\xi}_{\mathrm{req}} = \left(\mathbf{v}_{\mathrm{req}_n}^{\mathrm{T}}, \boldsymbol{\omega}_{\mathrm{req}_n}^{\mathrm{T}}\right)^{\mathrm{T}} = \left(\xi_{\mathrm{req}_1}, \xi_{\mathrm{req}_2}, \xi_{\mathrm{req}_3}, \xi_{\mathrm{req}_4}, \xi_{\mathrm{req}_5}, \xi_{\mathrm{req}_6}\right)^{\mathrm{T}}$ with some $\xi_{\mathrm{req}_i} = 0$ defined by the designer. The required DOF of the manipulator are the amount of $\xi_{\mathrm{req}_i} \neq 0$. For instance, the vector $\boldsymbol{\xi}_{\mathrm{req}} = \left(\xi_{\mathrm{req}_1}, \xi_{\mathrm{req}_2}, \xi_{\mathrm{req}_3}, 0, 0, \xi_{\mathrm{req}_6}\right)^{\mathrm{T}}$ defines the motion of a manipulator with 4DOF (1R3T) on the EE: three translational motions along the x, y and z-axis (3T) and one rotational motion around the z-axis (1R).

A mechanism to fulfil the required motion is a robot manipulator defined through the DH parameters. Its Jacobian matrix is $\mathbf{J} = (\mathbf{J}_1, \mathbf{J}_2, \ldots, \mathbf{J}_n)$ and the velocity of the EE is defined as in (1) through the robot motion vector $\boldsymbol{\xi} = (\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6)^{\mathrm{T}}$. To choose a mechanism as a suitable architecture, this must be verified with regard to the next conditions:

1. The terms of the required motion vector $\boldsymbol{\xi}_{\mathrm{req}}$ that are zero, must also be zero in the robot motion vector $\boldsymbol{\xi}$ of the given manipulator, i.e. if $\xi_{\mathrm{req}_i} = 0$ in $\boldsymbol{\xi}_{\mathrm{req}}$ then $\xi_i = 0$ in $\boldsymbol{\xi}$ for the same i.

2. The rank$(\mathbf{J})$ of the mechanism must be equal to the number of required DOF.

The disadvantage of using the DH parameters is that the axis of the first joint is always the z-axis and all configurations will have a motion along or around it. To avoid this problem and consider solutions in which the axis of the first joint is also aligned with the x and y-axis, the orientation of the axis of the first joint can be changed through the rotation matrices $\mathbf{R}_x$ and $\mathbf{R}_y$ defined as:

$$\mathbf{R}_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \quad \mathbf{R}_y = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \tag{4}$$

where $\mathbf{R}_x$ is obtained rotating $-\pi/2$ around the x-axis and $\mathbf{R}_y$ by rotating $\pi/2$ around the y-axis. Then, the matrices $\mathbf{R}'_x$ and $\mathbf{R}'_y$ can be formulated to change the orientation of the required motion $\boldsymbol{\xi}_{\text{req}}$:

$$\mathbf{R}'_x = \begin{pmatrix} \mathbf{R}_x^{\text{T}} & \mathbf{0}_{[3\times3]} \\ \mathbf{0}_{[3\times3]} & \mathbf{R}_x^{\text{T}} \end{pmatrix} \quad \mathbf{R}'_y = \begin{pmatrix} \mathbf{R}_y^{\text{T}} & \mathbf{0}_{[3\times3]} \\ \mathbf{0}_{[3\times3]} & \mathbf{R}_y^{\text{T}} \end{pmatrix}, \tag{5}$$

$$\boldsymbol{\xi}_{\text{req}}^{(z)} = \boldsymbol{\xi}_{\text{req}} , \tag{6}$$

$$\boldsymbol{\xi}_{\text{req}}^{(y)} = \mathbf{R}'_x \boldsymbol{\xi}_{\text{req}} , \tag{7}$$

$$\boldsymbol{\xi}_{\text{req}}^{(x)} = \mathbf{R}'_y \boldsymbol{\xi}_{\text{req}} , \tag{8}$$

In this way, the axis of the first joint of the manipulator that are solutions for $\boldsymbol{\xi}_{\text{req}}^{(z)}$ is aligned with the z-axis of the base reference frame (see Fig. 2(a)), likewise for $\boldsymbol{\xi}_{\text{req}}^{(y)}$ and $\boldsymbol{\xi}_{\text{req}}^{(x)}$ the axes of the first joint are aligned with the y-axis and the x-axis, respectively (see Fig. 2(b) and Fig. 2(c)). The union of the three sets of solutions constitutes the entire set of solutions for $\boldsymbol{\xi}_{\text{req}}$.

4 ARCHITECTURES GENERATION ALGORITHM

The procedure to find the manipulators that fulfil the two conditions mentioned in Sect. 3 is shown in Algorithm 1. The method is based on EM but taking into account the direction of the desired. This method uses symbolic computational tools and the description of a robot through the DH.

The architecture and the geometry of a serial manipulator suitable for being a solution of the structural synthesis problem are described by means of the DH parameters using only prismatic (P) and revolute (R) joints. To create all suitable architectures, each DH parameter is limited to have only two possible values: $\theta_i = \left\{0, \frac{\pi}{2}\right\}$, $d_i = \{0, d_i^*\}$, $a_i = \{0, a_i^*\}$, $\alpha_i = \left\{0, \frac{\pi}{2}\right\}$, where d_i^* and a_i^* are parameters different than zero. The type of a joint is indicated as R or P. In general, θ_i and α_i can have other values, but just in order to explain the algorithm, they are limited to the mentioned two values. All possible combinations of these values for the i-th link are generated by the procedure GenerateAllPossibleLinks and enumerated in Table 1. The j-th combination of the i-th link is written as $\text{DH}_{ij} = (\text{R/P}, \theta_i, d_i, a_i, \alpha_i)$ and the values for θ_i or d_i are replaced by the corresponding joint coordinate q_i in case of R and P joints, respectively.

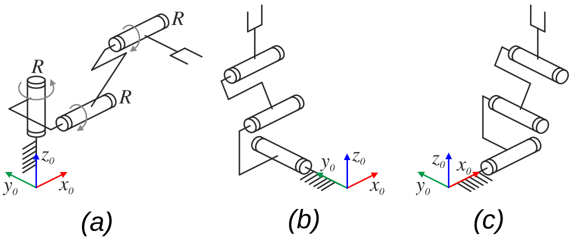

Fig. 2 Alignment of the first joint axis for a RRR robot with the (a) z-axis, (b) y-axis, and (c) x-axis.

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Input: ξ_req^(z)
ξ_req^(y) = R'_x ξ_req^(z);
ξ_req^(x) = R'_y ξ_req^(z);
foreach ξ_req ∈ (ξ_req^(z), ξ_req^(y), ξ_req^(x)) do
    for id = 1 to DOF do
        Links = GenerateAllPossibleLinks;
        if id = 1 then
            foreach L ∈ Links do
                ForwardKinematics(L);
                J = JacobianMatrix(L);
                ξ = J q̇;
                if ξ_i = 0 ∀ i = {1...n | ξ_req_i = 0} then
                    SetofCandidates = {SetofCandidates, L}
        else
            foreach Robot ∈ SetofCandidates do
                J = JacobianMatrix(Robot);
                ξ = J q̇;
                foreach L ∈ Links do
                    Robot* = AddLinkToRobot(L, Robot);
                    ForwardKinematics(Robot*);
                    J* = JacobianMatrix(Robot*);
                    ξ* = J* q̇;
                    if ξ*_i = 0 ∀ i = {1...n | ξ_req_i = 0} then
                        if rank(J*) = rank(J) + 1 then
                            SetofCandidates* =
                            {SetofCandidates*, Robot*}
            SetofCandidates = SetofCandidates *
    Save(SetOfCandidates)
```

Algorithm 1 Algorithm to find the solutions of the structural synthesis problem. L is each possible combination of DH parameters as shown in Table 1.

$\mathrm{DH}_{i1} = (\mathrm{P}, 0, q_i, 0,0)$	$\mathrm{DH}_{i9} = (\mathrm{R}, q_i, 0,0,0)$
$\mathrm{DH}_{i2} = (\mathrm{P}, 0, q_i, 0, \pi/2)$	$\mathrm{DH}_{i10} = (\mathrm{R}, q_i, 0,0, \pi/2)$
$\mathrm{DH}_{i3} = (\mathrm{P}, 0, q_i, a_i^*, 0)$	$\mathrm{DH}_{i11} = (\mathrm{R}, q_i, 0, a_i^*, 0)$
$\mathrm{DH}_{i4} = (\mathrm{P}, 0, q_i, a_i^*, \pi/2)$	$\mathrm{DH}_{i12} = (\mathrm{R}, q_i, 0, a_i^*, \pi/2)$
$\mathrm{DH}_{i5} = (\mathrm{P}, \pi/2, q_i, 0,0)$	$\mathrm{DH}_{i13} = (\mathrm{R}, q_i, d_i^*, 0,0)$
$\mathrm{DH}_{i6} = (\mathrm{P}, \pi/2, q_i, 0, \pi/2)$	$\mathrm{DH}_{i14} = (\mathrm{R}, q_i, d_i^*, 0, \pi/2)$
$\mathrm{DH}_{i7} = (\mathrm{P}, \pi/2, q_i, a_i^*, 0)$	$\mathrm{DH}_{i15} = (\mathrm{R}, q_i, d_i^*, a_i^*, 0)$
$\mathrm{DH}_{i8} = (\mathrm{P}, \pi/2, q_i, a_i^*, \pi/2)$	$\mathrm{DH}_{i16} = (\mathrm{R}, q_i, d_i^*, a_i^*, \pi/2)$

Table 1 List of possible DH parameter combinations for the i-th link

After that, the forward kinematics, the Jacobian matrix, and the velocity of the origin of the reference frame of each link are calculated through (1), (2), and (3). If the terms that are zero in the required motion vector $\boldsymbol{\xi}_{\mathrm{req}}$ are also zero in the robot motion vector $\boldsymbol{\xi}$ of the evaluated link, i.e. if $\xi_{\mathrm{req}_i} = 0$ and $\xi_i = 0$ for the same i, then this link is a candidate to become part of the solutions. If the link does not fulfil this condition, then it is discarded. At the end of this phase, there is a set of links which becomes a possible solution of the structural synthesis problem (SetofCandidates).

For the next stage, all possible configurations of the second link are generated again and added to each of the links chosen in the first stage. Each new manipulator is evaluated in the same way as in the first step and additionally, it is verified that the rank of the Jacobian matrix of each manipulator is greater than the rank of the manipulator in the previous stage. The manipulators that do not fulfil these two conditions are discarded for the next steps.

The next stages repeat the process of the second stage until the required DOF are achieved. The complete algorithm is carried out for the three required motion vectors $\boldsymbol{\xi}_{\mathrm{req}}^{(z)}$, $\boldsymbol{\xi}_{\mathrm{req}}^{(y)}$, and $\boldsymbol{\xi}_{\mathrm{req}}^{(x)}$. Since $\boldsymbol{\xi} \in \mathbb{R}^6$, the procedure is able to work up to 6DOF mechanisms.

As an example, finding all possible configurations with 3DOF (1R2T) that satisfy the required motion vector $\boldsymbol{\xi}_{\mathrm{req}} = \left(\xi_{\mathrm{req}_1}, \xi_{\mathrm{req}_2}, 0,0,0, \xi_{\mathrm{req}_6}\right)^{\mathrm{T}}$ is considered. The EE is able to move on a plane parallel to the xy-plane and rotate about an axis perpendicular to this. One of the solutions is shown in Fig. 3 and its DH parameters are shown in Table 2. This configuration is obtained by choosing links from Table 1 that fulfil the above mentioned conditions. By applying (2) and (3) to the first link (in this case rotational), we obtain (9). As can be seen, the 3rd, 4th, and 5th term of the vector $\boldsymbol{\xi}$ are zero as in the vector $\boldsymbol{\xi}_{\mathrm{req}}$ and, therefore, this link is chosen to continue in the synthesis process.

Link i	R/P	θ_i	d_i	a_i	α_i
1	R	q_1	d_1	0	$\pi/2$
2	P	0	q_2	0	$\pi/2$
3	R	q_3	d_3	0	0

Table 2 DH parameters of a manipulator with 3DOF (1R2T)

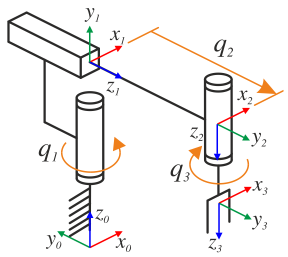

Fig. 3 3DOF (1R2T) manipulator

$$\boldsymbol{\xi} = \mathbf{J}\dot{\mathbf{q}} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \dot{q}_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \dot{q}_1 \end{pmatrix} \tag{9}$$

The second link is also generated in the same way and is joined with the first link to build a manipulator with two links whose velocity equation is shown in (10). The rank of the new Jacobian matrix is greater than the previous one and the 3rd, 4th, and 5th term of the vector $\boldsymbol{\xi}$ are still zero. For these two reasons the manipulator is considered for the next steps.

$$\begin{aligned} \boldsymbol{\xi} = \mathbf{J}\dot{\mathbf{q}} &= \begin{pmatrix} \cos(q_1)\, q_2 & \sin(q_1) \\ \sin(q_1)\, q_2 & -\cos(q_1) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} \\ &= \begin{pmatrix} \cos(q_1)\, q_2 \dot{q}_1 + \sin(q_1)\, \dot{q}_2 \\ \sin(q_1)\, q_2 \dot{q}_1 - \cos(q_1)\, \dot{q}_2 \\ 0 \\ 0 \\ 0 \\ \dot{q}_1 \end{pmatrix} \end{aligned} \tag{10}$$

Finally, one more link is added to build a 3DOF manipulator. Equation (11) shows the velocity equation of the manipulator. Now the vector $\boldsymbol{\xi}$ is the motion vector of the entire 3DOF robot whose 3rd, 4th, and 5th term are zero and the rank of the Jacobian matrix is 3. Hence the three columns $\mathbf{J}_1$, $\mathbf{J}_2$, $\mathbf{J}_3$ of the Jacobian matrix are linearly independent and the three required motions can be carried out independently.

$$\boldsymbol{\xi} = \mathbf{J}\dot{\mathbf{q}} = \begin{pmatrix} \cos(q_1)\, q_2 & \sin(q_1) & 0 \\ \sin(q_1)\, q_2 & -\cos(q_1) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{pmatrix} \tag{11}$$

$$= \begin{pmatrix} \cos(q_1)\, q_2\dot{q}_1 + \sin(q_1)\,\dot{q}_2 \\ \sin(q_1)\, q_2\dot{q}_1 - \cos(q_1)\,\dot{q}_2 \\ 0 \\ 0 \\ 0 \\ \dot{q}_1 - \dot{q}_3 \end{pmatrix} = \begin{pmatrix} \xi_{req_1} \\ \xi_{req_2} \\ 0 \\ 0 \\ 0 \\ \xi_{req_6} \end{pmatrix} = \boldsymbol{\xi}_{req} \,.$$

5 RESULTS OF THE PROPOSED APPROACH

The proposed algorithm was implemented in MAPLE17 and tested with several desired motions. The Tables 3 to 6 show the possible required motion vectors for manipulators with 2, 3, 4, and 5 DOF. In these tables all possible configurations for $\boldsymbol{\xi}_{req}$ are grouped according to the αRβT notation (and the number of solutions).

$\boldsymbol{\xi}_{req}^T$	N_S	$\boldsymbol{\xi}_{req}^T$	N_S
1R1T			
$(\xi_{req_1}, 0,0, \xi_{req_4}, 0,0)^T$	12	$(\xi_{req_1}, 0,0,0, \xi_{req_5}, 0)^T$	4
$(0, \xi_{req_2}, 0,0, \xi_{req_5}, 0)^T$	12	$*\,(\xi_{req_1}, 0,0,0,0, \xi_{req_6})^T$	4
$*\,(0,0, \xi_{req_3}, 0,0, \xi_{req_6})^T$	12	$(0, \xi_{req_2}, 0, \xi_{req_4}, 0,0)^T$	4
		$(0, \xi_{req_2}, 0,0,0, \xi_{req_6})^T$	4
		$(0,0, \xi_{req_3}, \xi_{req_4}, 0,0)^T$	4
		$(0,0, \xi_{req_3}, 0, \xi_{req_5}, 0)^T$	4
$*\,N_S^{(z)} = 12, N_S^{(y)} = 0, N_S^{(x)} = 0$		$*\,N_S^{(z)} = 0, N_S^{(y)} = 0, N_S^{(x)} = 4$	
2R		**2T**	
$(0,0,0, \xi_{req_4}, \xi_{req_5}, 0)^T$	12	$(\xi_{req_1}, \xi_{req_2}, 0,0,0,0)^T$	4
$(0,0,0, \xi_{req_4}, 0, \xi_{req_6})^T$	12	$*\,(\xi_{req_1}, 0, \xi_{req_3}, 0,0,0)^T$	4
$*\,(0,0,0,0, \xi_{req_5}, \xi_{req_6})^T$	12	$(0, \xi_{req_2}, \xi_{req_3}, 0,0,0)^T$	4
$*\,N_S^{(z)} = 0, N_S^{(y)} = 0, N_S^{(x)} = 0$		$*\,N_S^{(z)} = 0, N_S^{(y)} = 8, N_S^{(x)} = 8$	

Table 3 List of possible configurations for all possible requirements with 2DOF and number of solutions (N_S) found for each required motion vector

To explain the use of the tables, the second column of the 1R2T manipulators from Table 4 can be taken as an example. This group is composed by the manipulators that have three DOF in which the rotational direction is perpendicular to the translational directions: $*\,(\xi_{req_1}, \xi_{req_2}, 0,0,0, \xi_{req_6})^T$, $(\xi_{req_1}, 0, \xi_{req_3}, 0, \xi_{req_5}, 0)^T$, $(0, \xi_{req_2}, \xi_{req_3}, \xi_{req_4}, 0,0)^T$. For each required motion vector of this group, 240 different solutions were found. For the vector marked with the asterisk, the number of solutions are distributed as shown: 112 correspond to configurations whose first joint axis is parallel to the z-axis of the base reference frame ($N_S^{(z)}$). Likewise, there are 64 different configurations with the first joint axis parallel to the y-axis ($N_S^{(y)}$) and 64 further different configurations with the first joint axis parallel to the x-axis ($N_S^{(x)}$).

$\boldsymbol{\xi}_{req}^T$	N_S	$\boldsymbol{\xi}_{req}^T$	N_S
2R1T			
$*\,(\xi_{req_1}, 0,0, \xi_{req_4}, \xi_{req_5}, 0)^T$	0	$*\,(\xi_{req_1}, 0,0,0, \xi_{req_5}, \xi_{req_6})^T$	0
$(\xi_{req_1}, 0,0, \xi_{req_4}, 0, \xi_{req_6})^T$	0	$(0, \xi_{req_2}, 0, \xi_{req_4}, 0, \xi_{req_6})^T$	0
$(0, \xi_{req_2}, 0, \xi_{req_4}, \xi_{req_5}, 0)^T$	0	$(0,0, \xi_{req_3}, \xi_{req_4}, \xi_{req_5}, 0)^T$	0
$(0, \xi_{req_2}, 0,0, \xi_{req_5}, \xi_{req_6})^T$	0		
$(0,0, \xi_{req_3}, 0, \xi_{req_5}, \xi_{req_6})^T$	0		
$(0,0, \xi_{req_3}, \xi_{req_4}, 0, \xi_{req_6})^T$	0		
$*\,N_S^{(z)} = 0, N_S^{(y)} = 0, N_S^{(x)} = 0$		$*\,N_S^{(z)} = 0, N_S^{(y)} = 0, N_S^{(x)} = 0$	
1R2T			
$(\xi_{req_1}, \xi_{req_2}, 0, \xi_{req_4}, 0,0)^T$	32	$*\,(\xi_{req_1}, \xi_{req_2}, 0,0,0, \xi_{req_6})^T$	240
$(\xi_{req_1}, 0, \xi_{req_3}, \xi_{req_4}, 0,0)^T$	32	$(\xi_{req_1}, 0, \xi_{req_3}, 0, \xi_{req_5}, 0)^T$	240
$(\xi_{req_1}, \xi_{req_2}, 0,0, \xi_{req_5}, 0)^T$	32	$(0, \xi_{req_2}, \xi_{req_3}, \xi_{req_4}, 0,0)^T$	240
$(0, \xi_{req_2}, \xi_{req_3}, 0, \xi_{req_5}, 0)^T$	32		
$(\xi_{req_1}, 0, \xi_{req_3}, 0,0, \xi_{req_6})^T$	32		
$*\,(0, \xi_{req_2}, \xi_{req_3}, 0,0, \xi_{req_6})^T$	32		
$*\,N_S^{(z)} = 24, N_S^{(y)} = 0, N_S^{(x)} = 8$		$*\,N_S^{(z)} = 112, N_S^{(y)} = 64, N_S^{(x)} = 64$	
3R		**3T**	
$*\,(0,0,0, \xi_{req_4}, \xi_{req_5}, \xi_{req_6})^T$	6	$*\,(\xi_{req_1}, \xi_{req_2}, \xi_{req_3}, 0,0,0)^T$	96
$*\,N_S^{(z)} = 2, N_S^{(y)} = 2, N_S^{(x)} = 2$		$*\,N_S^{(z)} = 32, N_S^{(y)} = 32, N_S^{(x)} = 32$	

Table 4 List of possible configurations for all possible requirements with 3DOF and number of solutions (N_S) found for each required motion vector

$\boldsymbol{\xi}_{req}^T$	N_S	$\boldsymbol{\xi}_{req}^T$	N_S
3R1T		**1R3T**	
$(\xi_{req_1}, 0,0, \xi_{req_4}, \xi_{req_5}, \xi_{req_6})^T$	20	$*\,(\xi_{req_1}, \xi_{req_2}, \xi_{req_3}, 0,0, \xi_{req_6})^T$	3392
$(0, \xi_{req_2}, 0, \xi_{req_4}, \xi_{req_5}, \xi_{req_6})^T$	20	$(\xi_{req_1}, \xi_{req_2}, \xi_{req_3}, 0, \xi_{req_5}, 0)^T$	3392
$*\,(0,0, \xi_{req_3}, \xi_{req_4}, \xi_{req_5}, \xi_{req_6})^T$	20	$(\xi_{req_1}, \xi_{req_2}, \xi_{req_3}, \xi_{req_4}, 0,0)^T$	3392
$*\,N_S^{(z)} = 20, N_S^{(y)} = 0, N_S^{(x)} = 0$		$*\,N_S^{(z)} = 1984, N_S^{(y)} = 704, N_S^{(x)} = 704$	
2R2T			
$(\xi_{req_1}, \xi_{req_2}, 0, \xi_{req_4}, \xi_{req_5}, 0)^T$	0	$(\xi_{req_1}, \xi_{req_2}, 0, \xi_{req_4}, 0, \xi_{req_6})^T$	0
$(\xi_{req_1}, , \xi_{req_3}, \xi_{req_4}, 0, \xi_{req_6})^T$	0	$(\xi_{req_1}, \xi_{req_2}, 0,0, \xi_{req_5}, \xi_{req_6})^T$	0
$*\,(0, \xi_{req_2}, \xi_{req_3}, 0, \xi_{req_5}, \xi_{req_6})^T$	0	$(\xi_{req_1}, 0, \xi_{req_3}, \xi_{req_4}, \xi_{req_5}, 0)^T$	0
		$(\xi_{req_1}, 0, \xi_{req_3}, 0, \xi_{req_5}, \xi_{req_6})^T$	0
		$(0, \xi_{req_2}, \xi_{req_3}, \xi_{req_4}, \xi_{req_5}, 0)^T$	0
		$*\,(0, \xi_{req_2}, \xi_{req_3}, \xi_{req_4}, 0, \xi_{req_6})^T$	0
$*\,N_S^{(z)} = 0, N_S^{(y)} = 0, N_S^{(x)} = 0$		$*\,N_S^{(z)} = 0, N_S^{(y)} = 0, N_S^{(x)} = 0$	

Table 5 List of possible configurations for all possible requirements with 4DOF and number of solutions (N_S) found for each required motion vector

Since the direction of the rotational required degrees of freedom were defined related to the robot base reference frame, there are no results in the Tables 3 to 6 for architectures having two independent rotational degrees of freedom. It occurs because the turn around the first rotational joint affects the orientation of the second rotational joint. As an example we can take a 2DOF manipulator with 2 rotational joints as shown in Fig. 4 whose DH parameters are $(\theta_1 = q_1,\ d_1 = 0,\ a_1 = 0,\ \alpha_1 = \pi/2)$ for the first link and $(\theta_2 = q_2,\ d_2 = 0,\ a_2 = 0,\ \alpha_2 = 0)$ for the second link. The orientation of the joint axis $\mathbf{z}_0$ and $\mathbf{z}_1$ are defined using (3) and presented in (12). As can be seen, the direction of the second joint axis $\mathbf{z}_1$ is not constant. Therefore, the rotation directions are not independent and the mechanism would be discarded in the case that an architecture with two rotational degrees of freedom about fixed axes were required. This does not occur when the required rotational directions are defined related to the EE reference frame. Additional results considering this matter will be published subsequently.

2R3T	
$\boldsymbol{\xi}_{\mathrm{req}}^{\mathrm{T}}$	$\mathrm{N_S}$
$(\xi_{\mathrm{req}_1}, \xi_{\mathrm{req}_2}, \xi_{\mathrm{req}_3}, \xi_{\mathrm{req}_4}, \xi_{\mathrm{req}_5}, 0)^{\mathrm{T}}$	0
$(\xi_{\mathrm{req}_1}, \xi_{\mathrm{req}_2}, \xi_{\mathrm{req}_3}, \xi_{\mathrm{req}_4}, 0, \xi_{\mathrm{req}_6})^{\mathrm{T}}$	0
$*\ (\xi_{\mathrm{req}_1}, \xi_{\mathrm{req}_2}, \xi_{\mathrm{req}_3}, 0, \xi_{\mathrm{req}_5}, \xi_{\mathrm{req}_6})^{\mathrm{T}}$	0
$*\ \mathrm{N_S^{(z)}} = 0, \mathrm{N_S^{(y)}} = 0, \mathrm{N_S^{(x)}} = 0$	
3R2T	
$(\xi_{\mathrm{req}_1}, \xi_{\mathrm{req}_2}, 0, \xi_{\mathrm{req}_4}, \xi_{\mathrm{req}_5}, \xi_{\mathrm{req}_6})^{\mathrm{T}}$	828
$(\xi_{\mathrm{req}_1}, 0, \xi_{\mathrm{req}_3}, \xi_{\mathrm{req}_4}, \xi_{\mathrm{req}_5}, \xi_{\mathrm{req}_6})^{\mathrm{T}}$	828
$*\ (0, \xi_{\mathrm{req}_2}, \xi_{\mathrm{req}_3}, \xi_{\mathrm{req}_4}, \xi_{\mathrm{req}_5}, \xi_{\mathrm{req}_6})^{\mathrm{T}}$	828
$*\ \mathrm{N_S^{(z)}} = 220, \mathrm{N_S^{(y)}} = 220, \mathrm{N_S^{(x)}} = 388$	

Table 6 List of possible configurations for all possible requirements with 5DOF and number of solutions (N_S) found for each required motion vector

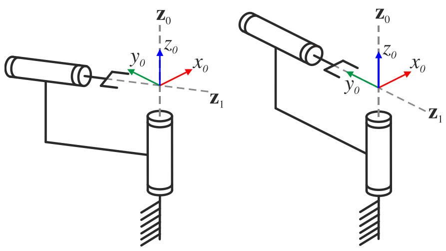

Fig. 4 2DOF manipulator

$$\boldsymbol{\xi} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \sin(q_1)\,\dot{q}_2 \\ -\cos(q_1)\,\dot{q}_2 \\ \dot{q}_1 \end{pmatrix} \qquad \mathbf{z}_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \mathbf{z}_1 = \begin{pmatrix} \sin(q_1) \\ -\cos(q_1) \\ 0 \end{pmatrix} \tag{12}$$

In order to facilitate the understanding of the results of the algorithm, as an example, the DH parameters of the 20 suitable architectures for a 4DOF with $\boldsymbol{\xi}_{\mathrm{req}} = (0,0,\xi_{\mathrm{req}_3}, \xi_{\mathrm{req}_4}, \xi_{\mathrm{req}_5}, \xi_{\mathrm{req}_6})^{\mathrm{T}}$, i.e. three rotational DOF and one translational motion along the z-axis, are shown in Table 7. Fig. 5 presents three distinctive examples of these architectures. In all three cases, the axes of the three R joints intersect in the same point.

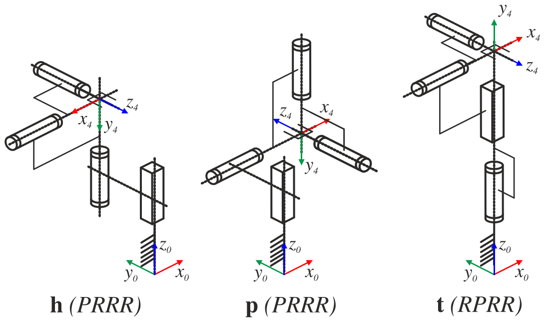

Fig. 5 4DOF (3R1T) manipulators with translational motion along the z-axis

6 CONCLUSIONS

In this paper, an approach for the automatic generation of serial robot architectures was presented. The method uses the rank of the Jacobian matrix as well as the velocity vector of the end effector to evaluate the fulfilment of the required motion on the direction of each axis of the robot base reference frame. With the proposed method, all suitable configurations can be generated in terms of symbolic DH parameters. This allows for using the created architectures for a structural synthesis process. Hence, it can be used in a robot computer aided design process evaluating the performance of each robot regarding requirements as workspace, dynamics, etc. Results of the proposed algorithm for manipulators up to five degrees of freedom are listed and the generated architectures for a 4DOF manipulator are given as an example.

Future work will deal with the detection of isomorphisms in the set of suitable architectures generated by this algorithm in order to find which DH parameters does not affect the DOF of the EE. In this way, the generated architectures can be grouped and the geometrical synthesis can be carried out for each group using the chosen parameters, thus, reducing the optimization problem.

7 ACKNOWLEDGMENT

Daniel Ramirez would like to thank Universidad Santo Tomas and also the Administrative Department of Science, Technology, and Innovation of Colombia Colciencias for financial support through the Call 529 2011.

PRRR					
$\begin{bmatrix} \mathrm{P} & 0 & q_1 & 0 & 0 \\ \mathrm{R} & q_2 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_3 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_4 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} \mathrm{P} & 0 & q_1 & 0 & 0 \\ \mathrm{R} & q_2 & d_2^* & 0 & \pi/2 \\ \mathrm{R} & q_3 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_4 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} \mathrm{P} & \pi/2 & q_1 & 0 & 0 \\ \mathrm{R} & q_2 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_3 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_4 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} \mathrm{P} & \pi/2 & q_1 & 0 & 0 \\ \mathrm{R} & q_2 & d_2^* & 0 & \pi/2 \\ \mathrm{R} & q_3 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_4 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} \mathrm{P} & 0 & q_1 & a_1^* & 0 \\ \mathrm{R} & q_2 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_3 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_4 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} \mathrm{P} & \pi/2 & q_1 & a_2^* & 0 \\ \mathrm{R} & q_2 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_3 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_4 & 0 & 0 & 0 \end{bmatrix}$
a	**b**	**c**	**d**	**e**	**f**
$\begin{bmatrix} \mathrm{P} & 0 & q_1 & a_1^* & 0 \\ \mathrm{R} & q_2 & d_2^* & 0 & \pi/2 \\ \mathrm{R} & q_3 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_4 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} \mathrm{P} & \pi/2 & q_1 & a_1^* & 0 \\ \mathrm{R} & q_2 & d_2^* & 0 & \pi/2 \\ \mathrm{R} & q_3 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_4 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} \mathrm{P} & 0 & q_1 & 0 & \pi/2 \\ \mathrm{R} & q_2 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_3 & d_1 & 0 & \pi/2 \\ \mathrm{R} & q_4 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} \mathrm{P} & \pi/2 & q_1 & 0 & \pi/2 \\ \mathrm{R} & q_2 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_3 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_4 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} \mathrm{P} & 0 & q_1 & 0 & \pi/2 \\ \mathrm{R} & q_2 & d_2^* & 0 & \pi/2 \\ \mathrm{R} & q_3 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_4 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} \mathrm{P} & \pi/2 & q_1 & 0 & \pi/2 \\ \mathrm{R} & q_2 & d_2^* & 0 & \pi/2 \\ \mathrm{R} & q_3 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_4 & 0 & 0 & 0 \end{bmatrix}$
g	**h**	**i**	**j**	**k**	**l**
$\begin{bmatrix} \mathrm{P} & 0 & q_1 & a_1^* & \pi/2 \\ \mathrm{R} & q_2 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_3 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_4 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} \mathrm{P} & \pi/2 & q_1 & a_1^* & \pi/2 \\ \mathrm{R} & q_2 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_3 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_4 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} \mathrm{P} & 0 & q_1 & a_1^* & \pi/2 \\ \mathrm{R} & q_2 & d_2^* & 0 & \pi/2 \\ \mathrm{R} & q_3 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_4 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} \mathrm{P} & \pi/2 & q_1 & a_1^* & \pi/2 \\ \mathrm{R} & q_2 & d_2^* & 0 & \pi/2 \\ \mathrm{R} & q_3 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_4 & 0 & 0 & 0 \end{bmatrix}$		
m	**n**	**o**	**p**		

RPRR			
$\begin{bmatrix} \mathrm{R} & q_1 & 0 & 0 & 0 \\ \mathrm{P} & 0 & q_2 & 0 & \pi/2 \\ \mathrm{R} & q_3 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_4 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} \mathrm{R} & q_1 & 0 & 0 & 0 \\ \mathrm{P} & \pi/2 & q_2 & 0 & \pi/2 \\ \mathrm{R} & q_3 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_4 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} \mathrm{R} & q_1 & d_1^* & 0 & 0 \\ \mathrm{P} & 0 & q_2 & 0 & \pi/2 \\ \mathrm{R} & q_3 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_4 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} \mathrm{R} & q_1 & d_1^* & 0 & 0 \\ \mathrm{P} & \pi/2 & q_2 & 0 & \pi/2 \\ \mathrm{R} & q_3 & 0 & 0 & \pi/2 \\ \mathrm{R} & q_4 & 0 & 0 & 0 \end{bmatrix}$
q	**r**	**s**	**t**

Table 7 DH parameters of the suitable architectures with three rotational DOF and one translational motion along the z-axis: $\boldsymbol{\xi}_{req} = \left(0,0,\xi_{req_3},\xi_{req_4},\xi_{req_5},\xi_{req_6}\right)^T$

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