

Dynamic Biomorphic Control of the Distributed Systems

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ABSTRACT

The report proposes an algorithm of biomorphic control of the distributed systems, which includes an integral weighted estimation of the data coming from sensors and feedback control by a combination of individual characteristic behavior of the system. Example of this approach is discussed.

1 INTRODUCTION

The main feature of distributed systems is a great or even theoretically infinite number of degrees of freedom in the equations of motion or state. This leads to a formal violation of the conditions of observability. Consequently, the controllability may be compromised. At least the synthesis of control signals by using optimal criteria requiring knowledge of all of the state vector is not possible.

There is another problem corresponded to the static "compliance" and dynamic delay in distributed systems. Due to the delay, which inevitably arises between different parts of the system, the stability of the system with negative feedback can be broken. Pure time delay or phase shift leads to significant limitations on the effective value of the open loop gain, which in turn limits the accuracy of the control. These features lead to the hard principle of designing of control systems for distributed objects. It consists in the fact that the feedback sensor located directly at the point of application the efforts or actuators or rigidly connected with them. This configuration leads to the limitation of the number of sensors. Further design includes, as a rule, the synthesis of optimal feedbacks using H_2 or H_∞ criteria [1-5].

The disadvantage of this approach is the greater complexity of the transfer functions and sensitivity of the closed loop settings to the parameters of the system and various types of noise in the operation of the sensor. In fact, the delay in the controlled object is replaced by the advance inside of the control system. This also leads to the limitation of the effective gain coefficient. Consequently, the error of control remains large.

Identification of the parameters of the object to tune of the controllers is also a problem. As rule advanced algorithms are sensitive to even small non-linearity of object. It should be noted that the above approaches have developed historically in part, due to the considerable price of the feedback sensors.

A different picture can be observed in biological objects. Each muscle is surrounded by hundreds of different sensors - nerve endings. An important feature of the biological of control systems is the ability to increase the number of sensors and the number of feedback loops during training due to the growth neural tissue. In this case, the number of muscle does not increase, and the possibility of a biological object implement these or those movements are greatly increased. Such an adaptive mechanism allows us to adapt the control system to any conditions. Should be further noted that the biological objects do not solve mathematical equations when tuning control algorithms.

Thus, the reproduction of biological control systems allows to obtain the robust system control of distributed objects.

2 THE BIOMORFIC DESIGN OF THE CONTROL

In contrast of the living nature we can not to increase the number feedback sensors in technical systems. At least, this process is very expensive. Advantages of biological control systems can be implemented due to initial redundancy of sensors [6-9]. This redundancy is harmful in the ordinary control, as the noise and inaccuracies in the sensor will result in a reaction of the controlled system to non-existent changing in observed state variables.

Thus, the basis of the biomorphic control is the redundancy of feedback sensors and regularization of their data on the basis of eigen functions of the object motion [10-12].

Let us consider a finite-dimensional model of a controlled mechanical object in the linear approximation. The equations of such an object have the form:

$$\begin{aligned}\dot{X} &= AX + BU + Gf \\ Y &= CX \\ X(0) &= X_0\end{aligned}\quad (1)$$

where X is state variable vector of the object having the dimension $[n]$, Y is the observation vector of the dimension $[m]$, U is the control vector of the dimension $[l]$, f is the vector of external uncontrolled forces of the dimension $[r]$, X_0 is the initial condition vector, A is the matrix of a linearized system of equations that describes the controlled object and has the dimensions $[n \times n]$, G is the matrix of external actions of the linearized system of equations with the dimensions $[n \times l]$, B is the control matrix of the linearized system of equations with the dimensions $[n \times r]$, and C is the observer matrix with the dimensions $[m \times n]$.

In these equations, we do not take into account the dynamics of electronic devices for data collection and acquisition and the dynamics of the power systems feeding the exciters. These assumptions are plausible since these systems function at frequencies which are higher than those of the mechanical part of the object.

The standard mechatronic approach [6] presumes that the feedback sensors are located at the places where the control actions are applied. Then, the numbers m and l coincide and one can construct m feedback loops and establish a control in each of them as follows:

$$U_i = -H_i Y_i \quad (2)$$

where U_i is the control vector component, Y_i is the observation vector component, and H_i is the operator of the control system in the i -th feedback loop. As a rule, the feedback loops employ PID controllers, i.e., in the space of Laplace variables $H_i(p)$ is a fractional rational function whose numerator and denominator are the second-order polynomials.

To control the object, we will use the decomposition approach [10-12]. The vibration modes of the motion of the elastic object define a nonsingular transformation matrix S such that the matrix

$$SAS^{-1} = \Lambda_A$$

is a diagonal one. Multiplying the right-hand and left hand sides of (1) by the matrix S and using the substitution $SX = q$ we obtain the system

$$\begin{aligned}\dot{q} &= SAS^{-1}q + SBU + SGf \\ Y &= CS^{-1}q \\ q(0) &= SX_0\end{aligned}\quad (3)$$

where q is the vector of the coefficients of the vibration modes of the motion of the elastic object with the dimension $[n]$, that replaces the vector of state variables. Determining vector q from the sensor data Y can present a problem, however we assume that the number of sensor is sufficient to our aim, i.e., the information matrix is not singular and, therefore, the reconstruction can be performed by inverting the matrix C as follows: $C^{-1} = (C^T C)^{-1} C^T$.

Then,

$$q = S(C^T C)^{-1} C^T Y \quad (4)$$

where $S(C^T C)^{-1} C^T$ is the matrix transforming the observation vectors to the coefficients of vibration modes of the elastic object.

The equations of system (3) are coupled, which makes it difficult to restrict the number of variables of the vector q and complicates the control with a varied number of variables. In the framework of the biomorphic approach we suggest the choice of a control in the form

$$U = kF(q^* - q) \quad (5)$$

where q^* is the external action on the mode coefficient; k is the gain factor, which, in the general case, can be an operator (e.g., a PID controller); and F is a matrix such that

$$SBF = \Lambda_B$$

has a diagonal structure.

The algorithm for choosing the matrix F will be considered below. Assume that this choice is possible. Then, from system (3), we obtain

$$\begin{aligned}\dot{q} &= \Lambda_A q + k\Lambda_B(q^* - q) + SGf \\ q &= S(C^T C)^{-1} C^T Y \\ q(0) &= SX_0\end{aligned}\quad (6)$$

The scheme of the biomorphic design is shown in Fig.1.

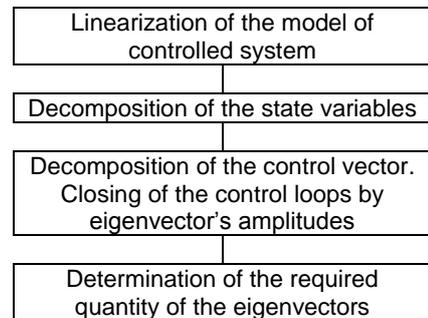


Fig. 1 Scheme of the biomorphic design

System (6) has a number of advantages listed in what follows.

1. All feedback loops are independent because, in each loop since the control is performed by each type of motion. Since the vibration modes of elastic bodies are orthogonal, the motion of any vibration mode does not affect the others. With account for the friction and nonlinearities, this independence is incomplete up to the friction forces, which, in the non-resonant modes, are small compared to the elastic and inertial forces. For this control the energy costs are minimal.

2. The number of feedback loops can vary. One can choose several vibration modes and control only these without spending resources and time of the control system to processing the feedback signals. It is evident that in the case of symmetric or skew symmetric loads, only half of the vibration modes will be needed. Moreover, the rigidity of flexural modes rapidly increases with their number; therefore, the description of motion of the elastic object includes only three to eight modes. In addition to the reduction in the amount of the information processes in the system, the noise immunity of the control system will increase because the disturbances will be averaged over a limited number of vibration modes.

3. In the process of control of the object, it becomes possible to dynamically increase the number of modes performing the control, e.g., by using the information of the value of error. In combination with the possibility of varying the gain factor k , this advantage makes it possible to refer to the control as biomorphic one.

4. It becomes possible to provide non-oscillatory motion for at least a finite number of vibration modes. The variations in the gain factor k have no natural upper bounds, even if the sensors and actuators are placed in different parts of the elastic object.

5. It can be assumed that the suggested control system will be insensitive to the accuracy of the accuracy in the vibration modes and components of the matrix F . If the number of sensors is sufficiently large, the sensitivity of sensors to errors and faults of a part of the sensors will be reduced dramatically because their signals will be averaged over the vibration modes.

6. There is a possibility to save on the processing of feedback signals if the standard actions are specified. For example, for the vibration suppression, a signal of the same frequency is generated. In this case, the control system can only follow a slow variation

in the phase and amplitude of this signal and it significantly reduces the requirements on its speed (the fast and slow motions are assumed to be separated).

In the framework of the biomorphic approach, a number of questions remain unresolved. They are as follows:

- 1) the identification of the matrix F that describes the control of actuators due to each mode;
- 2) the optimal ratio between the number of feedbacks or vibration modes taken into account in the control and gain factor k ;
- 3) the sensitivity of the biomorphic control to the nonlinearity in the model of the object.

3 THE IDENTIFICATION OF MATRIX F

Let us take matrix F in the form

$$F = B^T (BB^T)^{-1} S^{-1}. \quad (7)$$

The matrix (BB^T) is assumed non-singular due to a large number of actuators; the matrix S^{-1} exists by the definition of a nonsingular transformation. Then, the matrix

$$SBF = SBB^T (BB^T)^{-1} S^{-1}$$

is diagonal due to orthogonality of the eigenvectors forming the matrix S .

By the physical meaning, each column of matrix F is a combination of the control actions that produces a vibration mode in the controlled object, the number of the mode being equal to the number of the column. Due to the discrete character of the control actions and the limited number of actuators, the exact diagonalization for an arbitrary number of vibration modes is impossible. On the other hand, the algorithm of the control can employ any matrix F that provides stability of the control and the necessary accuracy of the results.

Let us consider an algorithm for constructing matrix F . Let us assume that, due to some vector of the control action U^1 , the object takes the position that corresponds to the vibration mode S_1 . In addition, we assume that the object does this rather slowly, i.e.,

$$\dot{X} \approx 0,$$

and external forces are absent (as it always happens in the learning). Then, the object equations imply that

$$AS_1 + BU^1 \approx 0 \quad (8)$$

By definition of the eigenvector

$$AS_1 = \lambda_A^1 S_1.$$

Then, vector

$$BU^1 = -\lambda_A^1 S_1$$

is orthogonal to all modes S_i except for the first one.

Hence, from the set of corresponding control vectors U^1, U^2, \dots, U^k , we can construct the matrix F that will make a diagonal matrix from the scalar product SBF . The algorithm for identifying matrix F is reduced to elaborating control signals that allow the object to reproduce its vibration modes, which can be done automatically without changing the physical structure of the links in the system being controlled. In this case, it is not necessary to use some special cumbersome procedures that require multiple multiplications or inversions of matrices, e.g., the recurrent least-square method.

4 BIOMORPHIC CONTROL OF THE KINEMATIC CHAIN

Let us consider the simplest example of a control of the kinematic chain.

The simplest model of the controlled kinematic chain is shown in Figure 2.

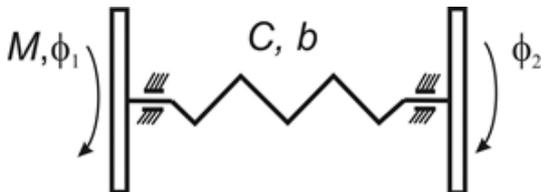


Fig. 2 Simplest model of the kinematic chain

Equations of motion are:

$$\begin{aligned} J_1 \ddot{\phi}_1 &= b(\dot{\phi}_2 - \dot{\phi}_1) + C(\phi_2 - \phi_1) + M \\ J_2 \ddot{\phi}_2 &= -b(\dot{\phi}_2 - \dot{\phi}_1) - C(\phi_2 - \phi_1) \end{aligned} \quad (9)$$

Here J_1, J_2 - the moments of inertia of the drive's rotor and kinematic chain (we assume it equal unit for simplicity), C and b - the rigidity and internal damping in the kinematic chain,, ϕ_1, ϕ_2 - the angles of rotation of the input and output parts of the kinematic chain, M - the moment acting from the drive side.

The objective of such systems is often acceleration and rotation with constant speed.

Typically, a feedback sensor which is used for this control, is mounted on the rotor drive. The general formula of such control has the form

$$M = k(\dot{\phi}^* - \dot{\phi}_1)$$

wherein k - gain $\dot{\phi}^*$ - the predetermined angular velocity. It should be noted that the condition of the controllability and the observability condition satisfied in this case. The whole system of equations has the form

$$\begin{aligned} \dot{X}_1 &= b(X_2 - X_1) + CX_3 + U \\ \dot{X}_2 &= -b(X_2 - X_1) - CX_3 \\ \dot{X}_3 &= X_2 - X_1 \\ U &= k(\dot{\phi}^* - X_1) \\ X(0) &= X_0 \end{aligned} \quad (10)$$

The position of the roots of the characteristic polynomial in the dependence of the gain is shown in the root locus in Figure 3.

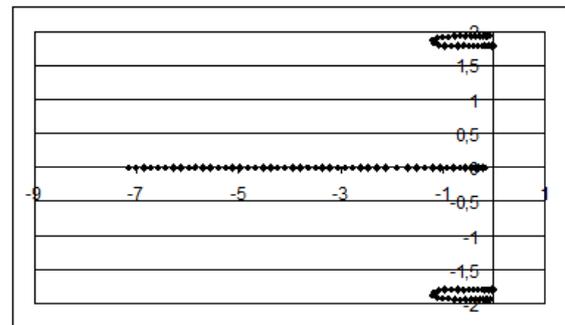


Fig. 3 Root locus.

The initial position of the roots is determined by the natural damping. Increasing the gain introduces an additional damping at the first, but then the natural frequency begins to decrease with decreasing of the damping. The damping with large gain becomes less natural. The point of maximum damping corresponds to the transition process, shown in Figure 4

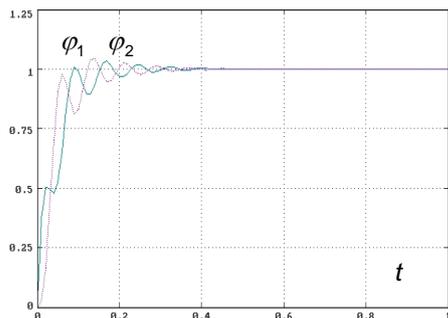


Fig. 4 The transition process

We compare obtained results with the proposed approach. We will consider only the data of the speed sensors. Obviously, the first natural shape will rotate as a rigid body, i.e. the vector $(1, 1)$, i.e. the vector $(1, -1)$.

Let the control is formed as

$$U = k_1(2\dot{\varphi}^* - (X_1 + X_2)) - k_2(X_1 - X_2)$$

$$\begin{aligned} q_1 &= X_1 + X_2 \\ q_2 &= X_1 - X_2 \end{aligned}$$

Root locus for case $k_2 = 0$ is shown in Figure 5

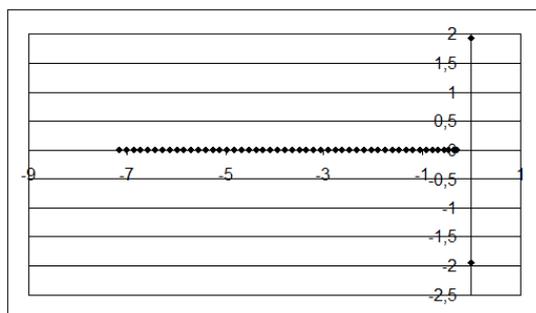


Fig. 5 Root locus for the case $k_2 = 0$.

The position of the roots with the imaginary part is not dependent on the gain k_1 . Hence, accuracy of stabilization speed (gain k_1) and vibration damping (gain k_2) may be divided.

Advantages of Biomorphic control are confirmed by the graphs of transients presented in Figures 6-8 for the optimum ratio of the gain coefficients, and for differing twice from the optimal values

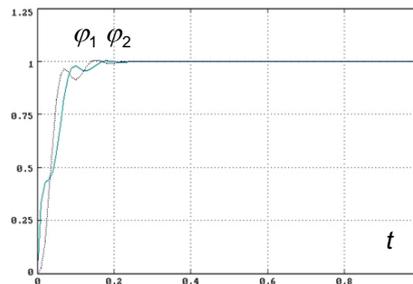


Fig. 6 The optimal transition process $k_1=25$, $k_2=50$

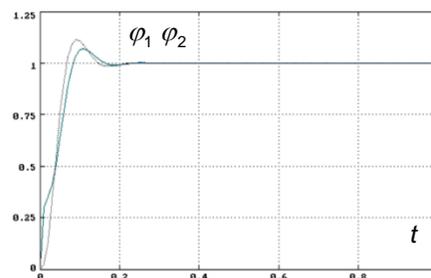


Fig. 7 The transition process $k_1=30$, $k_2=120$

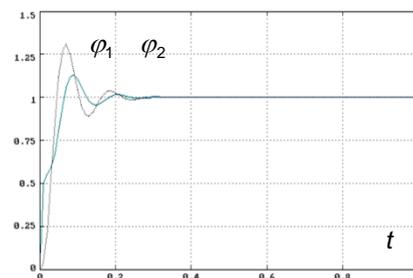


Fig. 8 The transition process $k_1=50$, $k_2=100$

Transient analysis shows that they are shorter and less oscillating than the standard process that is controlled by a single variable. In addition, there is the insensitivity of the transition process to change the settings of regulators. There is an opportunity to solve the problem of the accuracy and the task of suppressing vibrations separately.

5 CONCLUSIONS

The analysis of the results shows that the efficiency of biomorphic control is substantially higher than the efficiency of control with local feedbacks, the number of feedbacks being comparable and the gain factors being the same. The algorithm of biomorphic control does not require the exact identification of the matrix F . Thus, the algorithm proposed has a good robustness and admits the experimental

determination of the control action coefficients, which makes possible its efficient application in practice. The optimal ratio between the number of vibration modes taken into account in the biomorphic control and the gain factor requires additional study. Prescribing this relation will give additional possibilities for the active suppression of vibration.

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