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THE METHOD OF CONTROL SYSTEM SYNTHESIS FOR THE HEAT CONDUCTIVITY PROBLEM

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Abstract

If we regard controlled object as an object with distributed parameters, it will be possible to consider dependence between a regulating value (temperature, in the described case) and point of measuring and action in object volume. Generally, all physical agents are objects with distributed parameters. Control system theory is interested to research methods for realization a regulating function.

It is difficult to find a structure of control system for these objects, because there are no evident action points in all object volume, surface or length (3-D, 2-D or 1-D case, respectively). If we want to get a compound desired distribution, we should use a mobile power source. These restrictions lead us to necessary of not only action intensity regulation, but a position of action too.

To solve this problem there was made a discretization of distributed control object. This way object transformed in a quantity of interrelated objects (points or «object-points»). Each of these points is a control object with some inputs and outputs.

Firstly, it is an action input, an intensity of mobile power source influence on object by it. Secondly, it is a disturbance input, this input need to consider influence of another related neighbor object-points. And thirdly, temperature is a target parameter and also output of object. It is possible to use for these object-points a system structures from linear control system theory and methods of synthesis and analyses from it too. In the article described the method of synthesis temperature distribution control system for 1-D metal rod using specified admissions.

This method allows using of powerful instruments of linear control system theory for objects with distributed parameters. It makes easy to create a new systems, because we get an opportunity to escapes from difficult partial differential equations. Existence of mobile power source enables to make any compound distribution along a whole length of rod acting on every object-point separately.

INTRODUCTION

To solve problems which are concerned with influence on a parameter distribution (temperature, in our case) a new part of control theory that has a collection of analysis and synthesis methods was founded. But in this article we offer to use a numerical method of synthesis to go away from difficult mathematical expressions with adaptation of linear control theory methods. To explain general theses of proposed method a metal rod that has a one coordinate will be used (1-D case).

THE METHOD DEFINITION

The using of numerical methods to solve heat conductivity problem sets in applying of grid methods. They are used to produce a transition from a system of partial differential equations to a system of difference equations. There are some schemes of solving such systems: explicit, implicit, symmetric and three-layer [2]. A choice of scheme determinates by desired accuracy, stability of solving and calculating costs. The implicit scheme has been chosen because of its absolutely stability, the second order of accuracy and relatively small iteration of calculating.

Grid method realization

All grid methods share object on N points (object-points) by spatial coordinate. By viewpoint of control system theory, it is interesting to make a full-object control function by realization of N object-point functions.

This is the first step we would transform system (1) of partial differential equations to system (2) of difference equations.

The system of partial differential equations [1]:

$$\frac{\partial u(x,t)}{\partial t} = a\frac{\partial^2 u(x,t)}{\partial x^2} + q(x,t) \qquad (1.1)$$

$$\left.\frac{\partial u(x,t)}{\partial x}\right|_{x=0} - \alpha(u(0,t) - T_{EN}) = 0 \qquad (1.2)$$

$$\left.\frac{\partial u(x,t)}{\partial x}\right|_{x=L} - \alpha(u(L,t) - T_{EN}) = 0\,, \qquad (1.3)$$

where *u(x,t)* - function of temperature distribution; *q(x,t)* - function of mobile power source; *x* - spatial coordinate; *t* - time coordinate; *a* - heat conductivity coefficient; α - temperature conductivity coefficient; T_{EN} - environment temperature.

For system (2) form a grid:

$$\Omega_{x,t} = \{x_i = \Delta x \cdot i, i = 1..N; \tau_n = \Delta\tau \cdot n, n = 1..M\}\,,$$

and the system of difference equations has next view:

$$\frac{u_i^{n+1} - u_i^n}{\Delta\tau} = a\frac{u_{i-1}^{n+1} - 2u_i^{n+1} + u_{i+1}^{n+1}}{\Delta x^2} + q_i^{n+1}\,, \qquad (2.1)$$

$$\frac{u_0^{n+1} - u_1^n}{\Delta x} - \alpha(u_0^{n+1} - T_{EN}) = 0\,, \qquad (2.2)$$

$$\frac{u_N^{n+1} - u_{N-1}^n}{\Delta x} - \alpha(u_N^{n+1} - T_{EN}) = 0\,, \qquad (2.3)$$

where $u_i^n = u(x_i, t_n)$; *Δx* - step for spatial coordinate *x*; *Δτ* - step for time coordinate *t*.

Control system elements

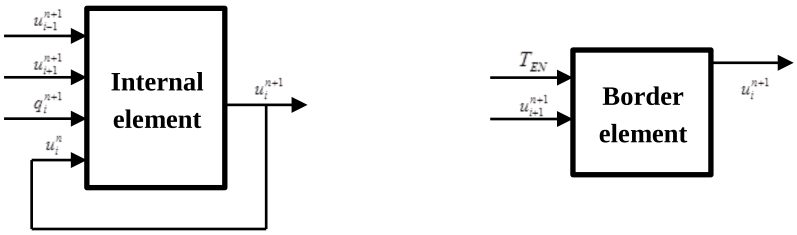
We could see two types of object-points from system (2):

1. Internal object or element (2.1);
2. Border object or element (2.2 and 2.3).

Firstly, we shall consider the internal element. Group equation by coefficient near u_i^n:

$$u_i^{n+1}(\frac{1}{\Delta\tau}+\frac{2a}{\Delta x^2})+u_i^{n+1}(-\frac{1}{\Delta\tau})+u_i^{n+1}(-\frac{a}{\Delta x^2})+u_i^{n+1}(-\frac{a}{\Delta x^2})+u_i^{n+1}(-1)=0\,. \quad (3)$$

Let's view this element as a graphic element (see fig. 1-a)



a. Internal element   b. Border element

Figure 1. Schematic view of object-points

The same way we could get a view of border element (see fig. 1-b).

Every element is a controlled object and the problem is to reach desired distribution (*$T_d(i)$, i=1,..,N*) by synthesis a control system realized an input influence q_i^n of mobile power source. To use terms of control system theory components u_{i-1}^{n+1} and u_{i+1}^{n+1} are disturbing influences. This way every internal element may be presented in control system as:

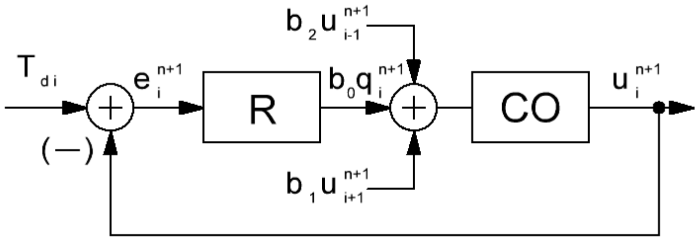


Figure 2. Control system structure

The control system should moves along rod to influence on distribution by power source.

Disturbance compensation

To get information about temperature in points *i, i-1* and *i+1* we should use a temperature sensor with three parts (three sensors).

We can't directly influence on u_{i-1}^{n+1} and u_{i+1}^{n+1}, this way we should compensate these influences by q_i^{n+1}. To reach this aim let's transform expression (3) to next view:

$$u_i^{n+1} = a_1 u_i^n + b_2 u_{i-1}^{n+1} + b_1 u_{i+1}^{n+1} + b_0 q_i^{n+1}, \qquad (4)$$

where:

$$a_1 = \frac{\Delta x^2}{\Delta x^2 + 2a\Delta\tau}; \quad b_2 = b_1 = \frac{a\Delta\tau}{\Delta x^2 + 2a\Delta\tau}; \quad b_0 = \frac{\Delta\tau\Delta x^2}{\Delta x^2 + 2a\Delta\tau}.$$

Expression (4) has two parts:

1. Itself object part:

   $U_i^n = a_1 u_i^n$;

2. Control influence part:

$$w_i^{n+1} = b_2 u_{i-1}^{n+1} + b_1 u_{i+1}^{n+1} + b_0 q_i^{n+1}. \qquad (5)$$

To analyze expression (5) we have the control influence as a sum of next actions:

- power supply action;
- "left" object-point action;
- "right" object-point action.

In this way we have a problem because on stage (at the time moment *n*) of forming of control influence we have no information about u_{i-1}^{n+1} and u_{i+1}^{n+1}. But to provide the implementation of (5) we could take into account large inertia of thermal processes and select $\Delta\tau \to 0$. Expression (5) will look this:

$$w_i^{n+1} = b_2 u_{i-1}^{n} + b_1 u_{i+1}^{n} + b_0 q_i^{n+1}. \qquad (6)$$

For time moment *n+1* u_{i-1}^{n} and u_{i+1}^{n} are constants.

To get a desired control action it is necessary to compensate u_{i-1}^{n} and u_{i+1}^{n} next way:

$$q_i^{n+1} = q_{i\_des}^{n+1} - \frac{b_2}{b_0} u_{i-1}^{n} - \frac{b_1}{b_0} u_{i+1}^{n}, \tag{7}$$

where $q_{i\_des}^{n+1}$ – designed control influence for isolated object-point. And we will get an equation of one object-point:

$$u_i^{n+1} = a_1 u_i^n + b_0 q_{i\_des}^{n+1}. \tag{8}$$

Expression (8) characterizes object-point as a one-order aperiodic element of linear control system. For this controlled object we could use methods of linear control system theory.

SUMMARY

To sum up it is necessary to say that the method had been presented makes it possible to use numerical methods and linear control theory methods to reach desired distribution for heat conductivity problems. There are some difficulties in realization with real (not model) physical agent:

- limited number of sensors;
- unknown coefficients, characterized controlled object.

To solve the first problem it is possible to use mobile temperature sensor with three sensors with three parts (*i-1, i, i+1*). To solve the second problem it is necessary to use parametric identification methods [3] of linear control theory.

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