

# Fully Automatic Parking Control System for Mobile Robot

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## Abstract

In this article the path planning principle for a prototype of four-wheeled vehicle for parallel and perpendicular parking maneuvers is considered. The curvilinear trajectory is formed based on geometrical calculations depending on the maximum possible steering angle and vehicle's size. The article presents a simplified mathematical description of the four-wheeled vehicle with steering control of Ackerman. The results were tested using a mobile platform (based on an electric quadrocycle).

## 1 INTRODUCTION

Recently the number of research in the area of autonomous vehicle control system is increased. In particular, autonomous driverless parking systems are taking an attention. This technology reduces the number of accidents on the road, while increasing the level of comfort for drivers (to choose parking lot and perform maneuver itself). Parallel parking is one of the most difficult maneuvers for entry-level driver, as it requires increased attention and driving experience.

Autonomous parking maneuver consists of three parts: the detection of free parking lot, the design of the necessary trajectory and estimation of the situation in the interaction with the environment.

To detect free space, heterogeneous sensors (ultrasonic, infrared, lidar and radar) and processing of images from cameras (computer vision) are used [1 - 4].

Approaches of trajectory motion planning include various techniques: geometric calculations with a constant radius [5], machine learning technologies [6] and neural networks [7]. The main disadvantage of the existing planning and tracking systems is the dependence of the system on the localization module. Such modules as GPS are inaccurate and its processing depends on different environment conditions. It can be adjusted by the introduction of additional sources of information (sensors and cameras).

Analytical methods based on the classical control theory are often used to control the movement and correct the obtained trajectory.

In this article, the planning path approaches for maneuvers of parallel and perpendicular parking are represented. These methods allow to use the minimum length of the free space and make movements along two arcs without a straight section between them (in the case of parallel parking) and along one arc (in the case of perpendicular parking).

## 2 KINEMATIC OF FOUR-WHEELED VEHICLE

In the construction of the mobile robot (control object) the same principles were implemented as in the construction of the full-drive car. Steering of the robot is carried out on the principle of Ackerman. The advantages of this solution include the ease of implementation of straight-line vehicle movement, and the main drawback is the inability to implement a turn along the central axle. The turning maneuver is always carried out in a circle, the minimum radius of which depends on the vehicle size and the maximum steering angle.

The most common mathematical description of a four-wheeled vehicle is the "bicycle model" (figure 1) [8], where the rear wheel is attached to the body and the front wheel can rotate about a vertical axis, thus allowing the control of the motion path. Some simplification were made: the body and wheels of the model are absolutely rigid, and it has no lateral inclination.

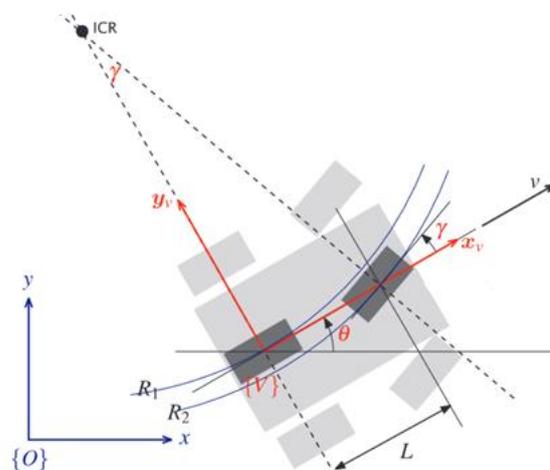


Fig. 1 «Bicycle model» of four-wheeled vehicle

This model is used as the description of wheeled vehicle movements in cases when the differences between the characteristics of the right and left wheel coupling between the axle and the road can be ignored. In addition, the bicycle model can be used for cases of car movement on a slippery surface with a sufficiently large track and speed, when the wheel coefficient of adhesion with the road is small, thus relatively small in absolute magnitude contact forces impact on car wheels [9].

The coordinate system  $\{V\}$  is connected to the car, the  $x_v$  axis is located in the center of the rear axle and shows the direction of movement. The position of the car in the coordinate system  $\{O\}$  can be represented by coordinates  $\alpha = (x, y, \theta)$ , where  $x$  and  $y$  – coordinates of the reference point of the coordinate system  $\{V\}$ , and  $\theta$  – the angle between the  $x$  and  $x_v$ .

The speed of the vehicle  $v$  is co-directional with the  $x$ -axis and is equal to 0 with respect to the  $y$ -axis (since the vehicle cannot move sideways):

$$\dot{x}^v = v; \dot{y}^v = 0. \quad (1)$$

The dashed lines in figure 1 show the directions in which the vehicle cannot move and their intersection is the instantaneous center of rotation.

The reference point is located in the center of the car rear axle and moves on a smaller radius  $R_1$ . Its angular speed is determined by the expression (2):

$$\dot{\theta} = \frac{v}{R_1}, \quad (2)$$

where  $v$  is the velocity of the vehicle.

Knowing the distance  $L$  between the front and rear axles of the car and steering angle  $\gamma$ , value of  $R_1$  can be calculated (3):

$$R_1 = \frac{L}{\operatorname{tg} \gamma}, \quad (3)$$

The turning radius is proportional to the length of the car. The steering angle  $\gamma$  is limited and its maximum value controls the minimum value of  $R_1$ .

Using equations (2) and (3), and the projection of the velocity vector of the vehicle on the axis of the coordinate system  $\{O\}$ , it is possible to obtain a system of equations describing the dependence of the coordinates and orientation of the vehicle from its speed and steering angle:

$$\begin{cases} \dot{x} = v \cdot \cos \theta; \\ \dot{y} = v \cdot \sin \theta; \\ \dot{\theta} = \frac{v}{L} \operatorname{tg} \gamma. \end{cases} \quad (4)$$

Consider the projection of the robot speed relative to the earth coordinate system on the  $y$ -axis of the coordinate system  $\{V\}$ :

$$\dot{y} \cos \theta - \dot{x} \sin \theta = 0. \quad (5)$$

The expression (5) is the non-holonomic constraint that the selected control object consists of, since the equation cannot be written in the form of relations between  $x, y, \theta$ . The non-holonomic constraint illustrates the fact that the vehicle cannot move sideways because the lateral velocity is 0, so the derivative constraint cannot be converted to a coordinate constraint.

### 3 PATH PLANNING MODULE

This module consist of two parts: parallel parking path planner and perpendicular parking path planner.

#### 3.1 PATH PLANNING WHILE PARALLEL PARKING

Consider an approach to plan the trajectory, which allows using the minimum length of the parking area and performing the movement along two arcs without a straight section between them. To obtain satisfying result the maximum possible steering angle is used (figure 2).

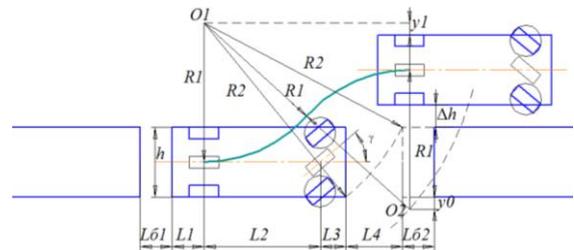


Fig. 2 Parallel parking trajectory

The steering angle  $\gamma$  is known, the radius of the circle  $R_1$  can be found by the formula (3), where  $L = L_2$  - the distance between the centers of the rear and front axles.

The condition of passability is the fact that the vehicle should not touch its front bumper the forward car (obstacle), i.e. the right front corner of the vehicle should follow along the arc radius  $R_2$ , defined by the expression (6):

$$R_2 = \sqrt{\left(R_1 + \frac{h}{2}\right)^2 + (L_2 + L_3)^2}, \quad (6)$$

where  $h$  is the width of the car;  $L_3$  is the distance from the front edge of the car to the axis of the driving wheels.

The vehicle should follow only two arcs without a straight section between them, so the center of rotation  $O_2$  should be located on the arc of double radius centered at  $O_1$ . In addition,  $\Delta h$  affects the location of the rotation center  $O_2$ . The greater the distance, the lower the center of rotation of  $O_2$ . Thus, while calculating the coordinates of the center  $O_2$ , it is possible to design the necessary trajectory for the maneuver. The coordinates of the center  $O_2(x, y)$  are presented as the formula (7):

$$\begin{aligned} x &= \sqrt{4R_1(h + \Delta h) - (h + \Delta h)^2}, \\ y &= 2R_1 \sin \varphi = 2R_1 - h - \Delta h, \end{aligned} \quad (7)$$

where  $y_1$  and  $y_0$  are presented by the formula (8):

$$y_1 = y_0 = R_1 - \frac{3h}{2} - \Delta h. \quad (8)$$

Based on the formula (9), the dependence of the coordinates of the rotation center  $O_2$  on  $\Delta h$  was established. In addition, affection of  $\Delta h$  on the rotation angle of the wheel was established. The dependence of the angle describing the arc on the value  $\Delta h$  is determined by the formula (9):

$$\varphi = \arcsin\left(\frac{2L_2 - h \cdot \text{th}\gamma - \Delta h \cdot \text{th}\gamma}{2L_2}\right). \quad (9)$$

Thus, it was found that the value  $\Delta h$  has an impact on such values as: the  $x$  and  $y$  coordinates of the  $O_2$  and the rotation angle for the arc, which play an important role in the designing of the vehicle trajectory.

The minimum length of the parking area includes safety zones – free space between parking vehicle and other cars (or obstacles), sizes of vehicle, as well as the value of the distance to the expected obstacles. To calculate the distance to the expected obstacle, the equality (10) can be used:

$$L_4 = 2R_1 \sin \varphi - L_2 - L_3. \quad (10)$$

The minimum distance  $S$  (11) to perform the parking maneuver is as follows:

$$S = L_{o1} + L_1 + L_2 + L_3 + L_4 + L_{o2}, \quad (11)$$

where  $L_1$  is the distance from the rear vehicle base to the axle of the driving wheels.

### 3.2 PATH PLANNING WHILE PERPENDICULAR PARKING

Consider the approach path planning, which allows using the minimum length of the parking area and performing the movement along the one arc. The maximum possible steering angle is used to obtain the satisfying result (figure 3). In figure 3,  $L_s$  are the safety zones,  $h$  is the width of the prototype vehicle,  $L_2$  is the distance between the front and rear vehicle axles,  $\Delta h$  is the distance to other vehicles (or obstacles).

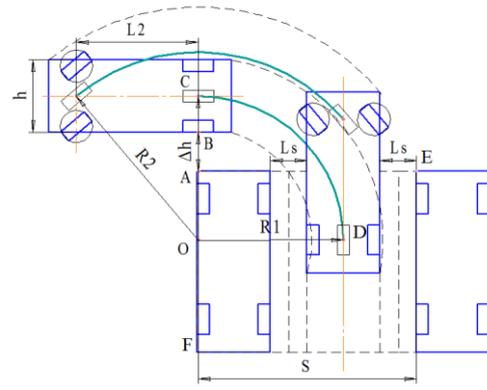


Fig. 3 Perpendicular parking trajectory

The condition of passability is fact that the car should not touch the corner of the car (obstacle) standing on the side (parallel) with its rear edge, i.e. in the "bicycle model" the averaged front wheel should follow along the arc with the radius  $R_2$  defined by the expression (12):

$$R_2 = \sqrt{R_1^2 + L_2^2}. \quad (12)$$

The length of this arc is determined by the formula (13):

$$L_d = \frac{R_2 \alpha \pi}{180}, \quad (13)$$

where  $\alpha$  in this case is  $90^\circ$ .

Once the vehicle is at point  $D$ , the front wheels should be aligned and the vehicle should go backwards the distance  $OF$  to ensure successful completion of the maneuver so as not to interfere with the movement of the remaining vehicles. The distance  $\Delta h$  is known, and therefore the segment  $OA$  will have the following length (14):

$$OA = R_1 - \frac{h}{2} - \Delta h, \quad (14)$$

The  $OF$  distance is as follow (15):

$$OF = L_2 + L_1 + L_3 - R_1 + \frac{h}{2} + \Delta h. \quad (15)$$

To determine the minimum length of the available area to maneuver,  $L_s$  safety zones were included, it will prevent possible collisions with other vehicles, then the minimum area length will be as follows (16):

$$S = R_1 + \frac{h}{2} + L_2 + L_s. \quad (16)$$

The depth of the "window" required for maneuver shall be equal to the length of the vehicle body. While designing perpendicular path planner, the following simplifications were made: in the process of maneuvering, the environment does not change; there is no slippage of the wheels; the maneuver is performed on a flat surface; the control system receives data about the steering angle and the path of the center of the rear axle, that can be calculated using rotation angle of the rear wheels differential body.

Thus, the connection between the coordinates of the center of rotation  $O$  and the value  $\Delta h$  was established, which allows to design the necessary trajectory that meets the conditions of the maneuver feasibility and the minimum value of parking area.

#### 4 CONCLUSION

In this article, two ways of path planning for a four-wheeled vehicle for performing parallel and perpendicular parking maneuvers were presented, respectively. These approaches allow maneuvering without a straight section between the arcs, which ensures the minimization of the available space. Despite the fact that presented approaches do not provide an absolute minimum of used space, the maneuver is performed without additional actions, which minimizes the time of the maneuver. Considering only maneuvers without extra-actions, proposed approaches provide the minimum of available space. The results will be used in competitions in the development of driverless vehicles.

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