EXPERIMENTAL STUDY OF LOCAL AND MODAL APPROACHES TO ACTIVE VIBRATION CONTROL OF ELASTIC SYSTEMS

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Abstract

The problem of control of distributed parameter systems is one of the important problems of the automatic control theory. Due to an infinite number of degrees of freedom, such systems are not observable and controllable. Two different methods, local and modal, are used to control distributed systems, and each of them has its own advantages and drawbacks. The aim of the present research is to implement and to compare experimentally these two methods for the problem of suppression of forced bending vibrations of a thin metal beam.

Two pairs of piezoelectric sensors and actuators were attached to the beam in each of the control systems considered. Their locations were chosen so as to provide the most efficient measurement and excitation of the first and the second vibration modes of the beam. Frequency methods of the automatic control theory were employed in order to design stable control systems that could efficiently suppress bending vibrations of the beam with the first and the second resonance frequencies. As a result, control systems with the desired performance were created based on both local and modal methods. The obtained modal system works efficiently for both resonances, while the local systems demonstrate appropriate performance either at the first or at the second resonance frequency only. This difference is due to the fact that in the case of modal control each control loop corresponds to a particular vibration mode and can be designed to provide optimal performance at the required frequency.

The performed research demonstrated the advantages of the modal method over the local one for the cases where it is necessary to suppress vibrations in the frequency range that contain more than one resonance frequency of the control object.
INTRODUCTION

Vibrations occur in a great variety of mechanical systems during their exploitation. In certain cases, these vibrations can be undesirable or even dangerous for the system and cause failure, damage or unwanted noise. The problem of vibration control is especially complicated in the case of elastic systems with distributed parameters, such as antennas, cables, robot manipulators, different beam and shell structures etc.

The absence of controllability and observability for distributed mechanical systems is due to an infinite number of eigenmodes, each of them being a separate degree of freedom. Of course, the modal stiffness is growing rapidly with increasing mode number, which allows one to neglect the higher modes of the object. However, the presence of higher modes leads to reduced accuracy and stability of the feedback control system. Moreover, the resonance behavior of the elastic object causes phase shifts which limit gain values in the control loops.

Traditionally for controlling distributed parameter systems the local method is used which implies that each sensor is connected to only one corresponding actuator located at the same region of the mechanical object. The phase shifts in this case are to be compensated by means of a sophisticated design of the feedback controllers. The methods of optimal control are widely used for designing controllers [1-5] as well as for determining the optimal spatial configuration of the control system (the mechatronic approach).

Alternatively, there is a modal method [6,7] based on the idea of independent control of different vibration modes of the mechanical system. This method allows one to compensate the phase shift for each mode in the corresponding controller, and it can provide stability of the system for the case of different spatial locations of sensors and actuators. At the same time, the modal system requires sufficient number of sensors and actuators to provide independent control of the modes.

The aim of the present research is to implement the both methods described above and to compare experimentally their efficiency for the problem of suppression of forced bending vibrations of a thin metal beam.

EXPERIMENTAL SET-UP

A control object is an aluminium beam 70 cm long with the cross-section 3×35 mm. It is disposed vertically and fixed at one point at the distance of 10 cm from the lower end. The external loading is a base excitation applied by means of a piezoelectric stack actuator which is a part of the fixation construction. The vibration of the stack actuator due to applied electrical signal causes bending vibration of the beam.

The control system includes two piezoelectric sensors and two actuators that are attached in pairs to the beam as it is shown in the figure 1. Once attached, they keep the same positions for all the control systems created. The actuators change the curvature of the beam depending on the control signal applied, while the sensors measure this curvature as the beam vibrates [8]. Actuators and sensors are connected through a controller.
The purpose of the control system is to suppress forced resonance vibrations of the beam with the first and the second resonance frequencies. These frequencies correspond to the first and the second bending modes of the beam. Therefore, sensors and actuators are attached to special locations on the beam, so that they can most efficiently measure and influence these particular modes.

![Fig.1. The scheme of the experiment](image)

In order to design local and modal control systems with the best performance frequency methods of the Automatic Control Theory are used. To monitor the efficiency of the control systems obtained the vibration amplitude of the upper endpoint of the beam is measured by the laser vibrometer. This choice is caused by the fact that the vibration amplitude of this point is the greatest one among all points of the beam for the vibration modes under consideration.

To work with the highest efficiency the piezoelectric sensors and actuators should be placed at the areas where the curvature of the modes to be controlled takes the maximum values. In order to satisfy this condition the first and the second bending modes of the beam were analyzed theoretically and experimentally (fig.2).

![Fig.2. The first and the second bending modes of the beam](image)
As a result, the location of the first pair of piezoelectric patches was defined to be near to the fixation point \((10.5 \, \text{cm} \leq x \leq 16.5 \, \text{cm})\), and the second pair was placed not far from the beam center \((37.5 \, \text{cm} \leq x \leq 43.5 \, \text{cm})\).

**LOCAL CONTROL SYSTEM**

The working scheme of the controller for the local control system is shown in the figure 3. Here \(y_1\) and \(y_2\) mean the signals measured by the sensors, while \(u_1\) and \(u_2\) are the control signals being sent to the actuators. In order to design such a system one needs to determine transfer functions \(R_1(s)\) and \(R_2(s)\), which establish the relation between measured and control signals.

\[
\begin{align*}
  &y_1 \\
  &\downarrow \quad R_1(s) \quad \uparrow \quad u_1 \\
  &y_2 \\
  &\downarrow \quad R_2(s) \quad \uparrow \quad u_2
\end{align*}
\]

*Fig.3. The scheme of the controller for the local control system*

**Design of the local control system**

To design the transfer functions for the controller the logarithmic characteristics of the open-loop system were analyzed [9-11]. The transfer function for each control loop was chosen in order to provide the most effective vibration suppression at the first and the second resonance frequencies of the beam. At the same time, special attention was paid to ensure stability of the systems obtained.

As a result two most effective local control systems were created. The first system shows the optimal performance at the first resonance frequency, while the second one gives the best result at the second resonance. Bode plots of the first and the second system are presented in the figures 4 and 5, respectively. The dashed curves represent the measured frequency response functions for the control object, and the solid curves correspond to the open-loop system.

*Fig.4. Bode plots for each control loop of the local control system #1*
Performance of the local control system

Frequency response functions in the vicinity of the first and the second resonance frequencies measured for the upper point of the beam are shown in the figure 6 for the first and the second local control systems, respectively. The black curve represents the uncontrolled system while the other curves correspond to the different gain values in control loops.

The first control system is effective at the first resonance, providing the decrease of the vibration amplitude by 77%, but it fails to suppress the second resonance. On the contrary, the second system has great performance at the second resonance (reduction of the amplitude is 88%), but at the first resonance it is not so good (only 45% reduction). The attempts to create a local system that could suppress both resonances with good efficiency were not successful.

MODAL CONTROL SYSTEM

The scheme of the controller for the local control system is shown in the figure 7. As before, $y_1$ and $y_2$ are the measured signals, while $u_1$ and $u_2$ are the control signals. In the control system all the sensors measure and all the actuators affect both first and second vibration modes of the beam, therefore the mode analyzer $T$ and the mode synthesizer $F$ are used to perform a linear transformation of signals in order to separate
the modes. To design a modal system one needs to define these matrices and to determine the transfer functions \( R_1(s) \) and \( R_2(s) \).

![Fig.7. The scheme of the controller for the modal control system](image)

**Design of the modal control system**

At the first stage of creating a modal system the mode analyzer and synthesizer are to be defined properly. As a result of this work two requirements should be observed: the first control loop should not affect and react to the activation of the second bending mode of the beam, and the second one should not affect and react to the activation of the first mode. The matrices \( T \) and \( F \) were first evaluated theoretically based on the known mode shapes and the measured frequency response functions of the control object, and then their values were specified precisely by performing additional experiments. The matrices obtained have the following form:

\[
T = \begin{pmatrix} 0.99 & 1.02 \\ -0.49 & 1.53 \end{pmatrix}, \quad F = \begin{pmatrix} 0.49 & -0.25 \\ 0.51 & 0.75 \end{pmatrix}
\]

(1)

Then the transfer functions are designed in order to provide the optimal performance of each modal control loop at the corresponding resonance frequency. Bode plots for each control loop of the system obtained are shown in the figure 8. The dashed curves represent the frequency response functions for the control object, and the solid curves correspond to the open-loop system.

![Fig.8. Bode plots for each control loop of the modal control system](image)

**Performance of the modal control system**

Frequency response functions in the vicinity of the first and the second resonance frequencies measured for the upper point of the beam for the created modal control
system are shown in the figure 9. The black curve represents the uncontrolled system while the other curves correspond to the different gain values in control loops.

![Figure 9. FRF of the modal system at the first and the second resonances](image)

The graphs show that the modal system is very effective at both resonance frequencies, providing the reduction of the vibration amplitudes by 84% and 87%, respectively.

**CONCLUSIONS**

The present research highlights application of the local and the modal approaches to active vibration control of distributed elastic systems was studied experimentally for the problem of suppression of bending vibrations of a metal beam. For both methods under consideration efficient control systems were created.

However, the local system failed to suppress efficiently vibrations at both resonances, showing the optimal performance either at the first or at the second resonance frequency. On the contrary, the local system succeeded in reducing vibrations at both resonance frequencies efficiently. Thus, the advantage of the modal approach over the local one was demonstrated for the cases where it is necessary to suppress vibrations in the frequency range containing multiple resonance frequencies of the control object. The result obtained can be explained by the fact that, in contrast to the local one, in the modal system each control loop corresponds to a particular vibration mode and can be designed to provide optimal performance at the corresponding resonance frequency.

**REFERENCES**


