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APPROXIMATE FEEDBACK LINEARIZING CONTROL OF NONLINEAR SINGULAR PERTURBED SYSTEMS

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Abstract

Singularly perturbed (SP) systems are used for description of various control objects that combine fast and slow motions. Multiple scales can be caused by different physical factors, such as the presence of small masses and moments of inertia, the high gain feedback etc. A mathematical description of such systems uses small parameters that multiply the time derivative of some state variables (fast state variables) in the equation of the system state. One of the most common methods of nonlinear control systems synthesis is the method of feedback linearization (FL). If the original nonlinear system cannot be linearized exactly by state feedback, the method of approximate feedback linearization (AFL) is used. The essence of AFL methods lies in the feedback linearization only of the some part of the original nonlinear system (not the entire system). In this paper, we propose a method of approximate feedback linearization control of nonlinear SP systems. Proposed method is based on the division of the original SP system and the construction of AFL control in the form of composite FL controls for slow and fast subsystems. The fast control is chosen so that the separating conditions (the Tikhonov theorem) were performed. Then, using a standard singular perturbation technique, we obtain a slow subsystem. Further the problem of FL control for slow subsystem is solved. Resulting AFL control is obtained in the form of composite control. The application of the proposed approach is illustrated through example.

INTRODUCTION

SP systems are used for description of various control objects that combine fast and slow motions. Multiple scales can be caused by different physical factors, such as the presence of small masses and moments of inertia, the high gain feedback etc (for example see [1] [2] [3] [4] [5] [6]). A mathematical description of such systems uses small parameters that multiply the time derivative of some state variables in the equation of the system state.

There are numerous publications devoted to the analysis and design of SP systems, which are reviewed in [1][7][8]. A recent review [8] includes more than 500 references and demonstrates a growing interest of researchers in nonlinear SP systems and their applications.

One of the most common methods of nonlinear control systems synthesis is the method of FL (external linearization) [9][10]. The idea of this method consists in converting the original nonlinear system into a linear one by means of feedback. Then, methods of control theory for linear systems are used for system design.

The problem of nonlinear SP systems control using FL is studied in [11][12][13][14]. In [11] a diffeomorphism is proposed that is independent of the small singular perturbation parameter. This diffeomorphism should satisfy the slow-fast dynamics separation condition. In [12] a new diffeomorphism is proposed, which does not require compliance with the dynamics separation conditions. Both of the above mentioned papers can be attributed to the same direct approach, which considers the linearization of the entire SP system.

Articles [13] [14] introduce another approach that is based on FL of the so-called "slow" subsystem, not the entire SP system. According to this indirect approach the original FL problem of nonlinear SP system is reduced to the simpler problem of feedback linearization of unperturbed slow subsystem. Restriction of the result in [13] [14] is the class of nonlinear SP systems that are linear in control input and fast state variables. In this paper the problem of FL is considered for the class of nonlinear SP system with nonlinear equation for slow state variables in general form and equation for fast state variables that is nonlinear only in slow state variables.

The paper is organized as follows: Section 2 contains the formulation of FL composite control problem for the class of SP systems described above; Section 3 discusses the decomposition process of nonlinear SP system using standard singular perturbation technique; Section 4 is devoted to the problem of synthesis of FL control for slow and fast subsystems; an example of the stabilizing control system development for a not full-state feedback linearizable system is shown in Section 5; conclusion with the main findings and acknowledgements are shown in Section 6 and Section 7 respectively.

PROBLEM FORMULATION

Consider a nonlinear SP system of the type

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2, u),\ \ x_1(0) = x_1^0, \\ \varepsilon \cdot \dot{x}_2 &= f_{21}(x_1) + f_{22}(x_1)x_2 + g_2(x_1)u,\ \ x_2(0) = x_2^0, \end{aligned} \tag{1}$$

where $x_1 \in R^{n_1}$, $x_2 \in R^{n_2}$ are the state variables vectors, $u \in R^1$ is the scalar control input, $\varepsilon > 0$ is a small parameter (singular perturbation).

It is assumed that the system (1) satisfies the following assumptions:

Assumption 1. The functions $f_1(x_1, x_2, u)$, $f_{21}(x_1)$, $f_{22}(x_1)$ and $g_2(x_1)$ are uniformly continuous and bounded, with a sufficient number of derivatives in their arguments.

Assumption 2. The function $f_{22}(x_1)^{-1}$ exists for all $x_1 \in D \subset R^{n_1}$.

It is necessary to transform SP system (1) to the "block triangular" form

$$\begin{aligned} \dot{z}_1(t) &= F_1(z_1, u),\ z_1(0) = z_1^0, \\ \dot{z}_2(\tau) &= F_2(z_1, z_2, u),\ z_2(0) = z_2^0,\ \tau = t/\varepsilon, \end{aligned} \tag{2}$$

where the first equation is independent of the fast state variables vector z_2. The form of the functions $F_1(z_1, u)$ and $F_2(z_1, z_2, u)$ will be determined later.

After the conversion of system (1) to form (2) the feedback linearization problem should be solved separately for the slow subsystem that is described by the first equation of system (2) and for the fast subsystem that is described by the second equation of system (2). The resulting FL composite control for the original SP system (1) is defined as the sum of slow and fast controls

$$u = u_s + u_f, \tag{3}$$

where u_s and u_f – control signals, obtained by solving the feedback linearization problem for slow and fast subsystems respectively.

DECOMPOSITION OF THE SYSTEM

Conversion of SP system (1) to the "block triangular" form (2) is carried out using standard singular perturbation technique [8] [9]. Letting $\varepsilon = 0$ in system (1) and solving the second equation of obtained reduced system with respect to x_2, by assumption 1 and 2 we have

$$x_2 = h_0(x_1, u_s) = -f_{22}(x_1)^{-1}(f_{21}(x_1) + g_2(x_1)u_s). \tag{4}$$

Substituting (4) into the first equation of system (1), we obtain an expression for the slow subsystem (via the change of variables $x_1 = z_1$)

$$\dot{z}_1(t) = f_1(z_1, h_0(z_1, u_s), u_s) = F_1(z_1, u_s),\ z_1(0) = z_1^0 = x_1^0. \tag{5}$$

To obtain the equation of the fast subsystem we represent the second equation of system (1) in fast time scale, replacing $\tau = t/\varepsilon$:

$$\dot{x}_2(\tau) = f_{21}(x_1) + f_{22}(x_1)x_2 + g_2(x_1)u,\ \ x_2(0) = x_2^0, \tag{6}$$

Substituting into (6) the expression for the composite control (3) we get the fast subsystem

$$\dot{x}_2(\tau) = f_{21}(x_1) + f_{22}(x_1)x_2 + g_2(x_1)(u_s + u_f).$$

Then applying the change of variables

$$x_1 = z_1,\ \ z_2 = x_2 + f_{22}(z_1)^{-1}(f_{21}(z_1) + g_2(z_1)u_s),$$

we finally have

$$\dot{z}_2(\tau) = f_{22}(z_1)z_2 + g_2(z_1)u_f,\ \ \ z_2(0) = z_2^0 = x_2^0 - h_0(z_1^0, u_s(0)). \tag{7}$$

In equation (7) slow state variables vector z_1 is treated as a fixed parameter [15] [16].

FEEDBACK LINEARIZATION

Before considering the problem of feedback linearization, we give the following basic notation [9], [10].

Let $\varphi(x)$ be a smooth function and $f(x)$ be a smooth vector field defined on $X \subset R^n$. The scalar function introduced as $L_f\varphi(x) = \dfrac{\partial\varphi}{\partial x} f(x)$ is so-called a (scalar) Lie derivative of scalar function $\varphi(x)$ along $f(x)$.

Let $f(x)$ and $g(x)$ be smooth vector fields defined on X. The vector field introduced as $L_f g(x) = \dfrac{\partial g}{\partial x} f(x) - \dfrac{\partial f}{\partial x} g(x),$ is a vector Lie derivative, often called a Lie bracket $[f(x), g(x)] = L_f g(x).$

In addition, the following notation is used to define the Lie derivative of order k:

$$L_f^0\varphi(x) = \varphi(x),\ \ L_f^0 g(x) = g(x),\ L_g L_f = L_g(L_f),\ \ \ L_f^k = L_f(L_f^{k-1}),\ k = 2,3,\ldots\ .$$

Feedback linearization of slow subsystem

Consider the slow subsystem

$$\dot{z}_1(t) = F_1(z_1, u_s),\ z_1(0) = z_1^0. \tag{8}$$

Let there exist a scalar function $\varphi(z_1)$. We define the Lie derivative of the function $\varphi(z_1)$ along the vector field $F_1(z_1, u_s)$ as $L_{F_1}\varphi(z_1, u_s) = \dfrac{\partial\varphi(z_1)}{\partial z_1} F_1(z_1, u_s).$ For the Lie derivatives of higher orders we have $L_{F_1}^k\varphi(z_1, u_s) = \dfrac{\partial L_{F_1}^{k-1}\varphi(z_1, u_s)}{\partial z_1} F_1(z_1, u_s),$ where $L_{F_1}^0\varphi(z_1, u_s) = \varphi(z_1).$

Assumption 3. The system (8) has a relative degree $r = n_1$ at the point (z_1^0, u_s^0), that is the next conditions hold:

1) $\frac{\partial}{\partial u_s} L_{F_1}^k \varphi(z_1, u_s) = 0$ for all z_1 in a neighborhood of z_1^0, all u_s in a neighborhood u_s^0 and all $k < r$.

2) $\frac{\partial}{\partial u_s} L_{F_1}^r \varphi(z_1^0, u_s^0) \neq 0$.

Due to the above conditions the first r Lie derivatives do not depend explicitly on the input: $L_{F_1}^k \varphi(z_1, u_s) = L_{F_1}^k \varphi(z_1),\ 0 \le k \le r-1$. Under the above assumptions, there exists a diffeomorphism [17]

$$T_s(z_1) = \{T_s^k(z_1)\},\ T_s^k(z_1) = L_{F_1}^{k-1}\varphi(z_1),\ 1 \le k \le n_1.$$

We define the vector ξ with elements expressed in terms of the transformation

$$\xi_1 = T_s^1(z_1) = L_{F_1}^0 \varphi(z_1) = \varphi(z_1),$$

$$\xi_k = \dot{\xi}_{k-1} = T_s^k(z_1) = L_{F_1}^{k-1}\varphi(z_1),\ 2 \le k \le n_1.$$

Differentiating with respect to time t the variable ξ_{n_1}, we obtain

$$\dot{\xi}_{n_1} = \vartheta(\xi, u_s) = L_{F_1}^{n_1}\varphi(T_s^{-1}(\xi), u_s).$$

The FL control law for the slow subsystem (8) is obtained by solving the following algebraic equation with respect to u_s

$$\vartheta(\xi, u_s) = \upsilon_s. \qquad (9)$$

If the equation (9) has an analytic solution $u_s = \psi(\xi, \upsilon_s)$, then the diffeomorphism $T_s(z_1)$ and the control law $u_s = \psi(\xi, \upsilon_s)$ transform the system (8) to the linear form $\dot{\xi}_1 = \xi_2,\ \xi_1 = \varphi(z_1),\ \dot{\xi}_2 = \xi_3, \ldots, \dot{\xi}_{n_1} = \upsilon_s$.

Feedback linearization of fast subsystem

Consider the fast subsystem

$$\dot{z}_2(\tau) = f_{22}(z_1)z_2 + g_2(z_1)u_f,\ z_2(0) = z_2^0 = x_2^0 - h_0(z_1^0, u_s(0)). \qquad (10)$$

In this equation the slow state variables vector z_1 is treated as a fixed parameter [15] [16].

As the system (10) is linear in the fast variables state vector z_2 and the control u_f, the external linearization (via feedback) is not required. In this situation, the fast control u_f is selected as a feedback (here $x_1 = z_1$)

$$u_f = -G_f(x_1)z_2 = -G_f(x_1)x_2 - G_f(x_1) f_{22}(x_1)^{-1}(f_{21}(x_1) + g_2(x_1)u_s), \qquad (11)$$

where $G_f(x_1)$ is designed such that $\mathrm{Re}\lambda\left(f_{22}(z_1) - g_2(z_1)G_f(z_1)\right) < 0,\ \forall z_1 \in D.$

The existence of $G_f(x_1)$ satisfying the last inequality is guaranteed if the pair $(f_{22}(z_1), g_2(z_1))$ is controllable uniformly in z_1, i.e. the corresponding controllability Grammian is bounded from below by a positive-definite matrix.

Feedback linearizing composite control

The resulting composite control is determined from (3) by substituting the relations for the fast and slow control:

$$u = \left(1 - G_f(x_1) f_{22}(x_1)^{-1} g_2(x_1)\right) u_s - G_f(x_1)\left(x_2 + f_{22}(x_1)^{-1} f_{21}(x_1)\right). \qquad (12)$$

It should be noted that the FL composite control (12) does not depend on a small parameter ε. The singular perturbation parameter ε can be considered as an uncertainty in the original system (1) [18]. Moreover, there exists some $\varepsilon^* > 0$ such that $\forall \varepsilon$: $0 < \varepsilon \le \varepsilon^*$ the original SP system (1) is stable, if the slow and the fast subsystems are stable. In this sense, the SP system (1) with the composite control (12) is robustly stable with respect to ε.

ILLUSTRATIVE EXAMPLE

Consider the system as follows

$$\begin{aligned} \dot{x}_1 &= x_3 - x_2, \\ \dot{x}_2 &= -x_2 - u, \\ \dot{x}_3 &= x_1^2 - x_3 + u, \end{aligned} \qquad (13)$$

with output $h(x) = x_1$. The system (13) has relative degree 2 for all x, therefore, it is not full-state feedback linearizable.

We apply the developed method. We introduce in (13), the ε parameter fictitiously. Let x_1 is a slow variable, x_2 and x_3 is fast variables. Setting $\varepsilon \to 0$, we obtain the slow subsystem

$$\dot{x}_s = x_s^2 + 2u_s, \qquad (14)$$

and the fast subsystem

$$\begin{aligned} \dot{x}_{f2} &= -x_{f2} - u_f, \\ \dot{x}_{f3} &= -x_{f3} + u_f. \end{aligned} \qquad (15)$$

Solving the problem of synthesis of FL control for the slow subsystem (13), we obtain

$$u_s = -0.5x_1^2 + \upsilon_s.$$

External control signal υ_s is convenient to implement in the form of proportional controller for tracking error minimization, ie

$$\upsilon_s = k_p e, \; e = x_{1ref} - x_1.$$

The fast control u_f for system (15) according to (11) is presented in the form

$$u_f = (G_{f3} - G_{f2})u_s - G_{f2}x_2 - G_{f3}(x_3 - x_1^2),$$

where G_{f2}, G_{f3} are feedback coefficients.

The resulting composite FL control according to (12) is equal to

$$u = (1 - G_{f2} + G_{f3})(-0.5x_1^2 + \upsilon_s) - G_{f2}x_2 - G_{f3}(x_3 - x_1^2). \qquad (16)$$

The simulation results of the system (13) with the composite FL controller (16) for $G_{f2} = -10^4, G_{f3} = 10^4$ и $k_p = 10$ and $\varepsilon = 1$ are shown on fig. 2. The reference signal is taken in the form of rectangular pulses with an amplitude of 2 and a frequency of 1 Hz. The fig.2 also shows the transient response for variable x_1 at $\varepsilon = 500$ (dotted line). The results show that the system is well work out a reference signal, wherein is robust with respect to the parameter ε, which was introduced fictitiously.

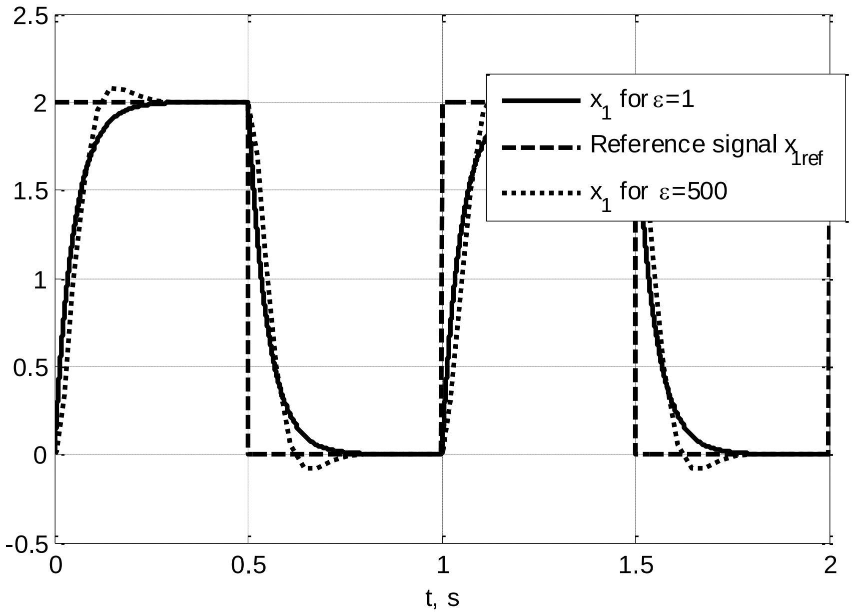

Fig.1. Transient response for variable x_1 at different values of ε

SUMMARY

In this paper, based on the idea of a composite control for a class of nonlinear SP systems we proposed a method for the synthesis of control by using of FL technique.

The advantage of the proposed method is to simplify the original FL problem for nonlinear SP system by its decomposition into two simpler problems: for the slow subsystem and for the fast subsystem. This fact is especially useful when the original nonlinear system is non-linearized via feedback. In this case the method of approximate FL is used [19] [20]. If the original non-linearized system can be represented in the SP form, the proposed method can be used as a means of synthesis of approximate FL control. The efficiency of the proposed method is demonstrated by an example.

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