Adaptive Control System with Parameter Tuning for Double Mass Elastic Electromechanical System

K. V. Ignatev

ABSTRACT

The paper describes design of adaptive control system with parameter tuning with reference model for linear double mass elastic electromechanical system. The system consists of dc motor connected via elastic link to metal disc. In order to suppress of elastic oscillations in linearized model of the system a Full State Feedback (FSF) controller was developed. In order to deal with parameter uncertainty an adaptive control system with reference model was added to the system. An adaptive control system tuning is based on solution of Lyapunov equation with symmetric positive definite matrix G. In most cases G is chosen to be a diagonal matrix with positive diagonal elements. This solution guarantees a stability of control system, but it is difficult to find good elements of matrix G to get better step response of the system with different sets of system parameters. A genetic algorithm was used in order to find elements of G matrix for better step response of the system. Mean difference between system step response and reference model output with different sets of system parameters is used as objective function for genetic algorithm. Linearized mathematical model of the system has 4 state variables: motor speed, elasticity torque, metal disc speed and metal disc angle. In order to acquire all state variables a Luenberger observer was added to the system.

1 INTRODUCTION

Mechatronic systems which include electromechanical objects with extended geometry and elastic deformations are widespread in modern industry. These systems include highly precision cutting machines, manipulators, high speed terrestrial and marine objects, etc. The main factor affecting their effectiveness is the elastic deformation of the units. Lower natural frequencies are normally in the range of 2-15 Hz, thereby, the danger of the excitation of elastic vibrations does not allow implementing high speed control. This leads to the need to reduce speed in such systems. In addition to the elastic deformation, the complexity of the control task is also increasing due to the parametric uncertainty of the system, as well as the inability to measure all of the state variables of the system. This article discusses the design of adaptive control system with parameter tuning and a full order state observer for the two-mass electromechanical elastic system and tuning it with genetic algorithms.

2 MATHEMATICAL MODEL

Consider the linearized mathematical model of electromechanical elastic two-mass system with two-loop control system. Neglect the electromagnetic inertia of the motor and consider control system with outer second mass position loop and inner motor speed loop.

$$\begin{cases} \dot{q}_2 = \omega_2 \\ \dot{\omega}_2 = \frac{\tau_e}{I_2} \\ \dot{\tau}_e = p(\omega_1 - \omega_2) \\ \dot{\omega}_1 = \frac{k_m k_y k_p b_s b_p}{I_1 R} u_c - \frac{k_m k_y k_p b_s b_p}{I_1 R} q_2 - \\ \quad \frac{k_m(k_e + k_c k_y b_c)}{I_1 R} \omega_1 - \frac{1}{I_1} \tau_e \\ u_c = u_0 + u_l + u_a \end{cases} \tag{1}$$

where q_2 – second mass angular position, ω_2 – second mass angular velocity, τ_e – elasticity torque, I_2 – second mass moment of inertia, p – link stiffness, ω_1 – motor angular velocity, I_1 – motor (first mass) moment of inertia, R – resistance of motor armature winding, k_e – motor back EMF constant, k_m – motor size constant, k_y – amplifier gain, b_p – second mass position loop gain, b_s – motor speed loop gain, k_p – second mass position sensor feedback gain, k_s – motor speed sensor feedback gain, u_c – total control action, $u_0 = b\varphi$ – control set point, b – the proportionality factor that is equal to the total feedback gain of controllable variable, φ – desirable angular position of second mass, u_l – full state feedback (FSF) control action, u_a – adaptive control action.

Let us take a system with the following parameters: k_m = 1 V·s, k_e = 1 V·s/rad, k_y = 22, k_s = 1 V·s/rad, k_p = 1 V/rad, I_1 = 0.05 kg·m^2, I_2 = 0.1 kg·m^2, R = 2 Ω, p = 25 N·m/rad. Proportionality factor b = k_p in the system without FSF and adaptive control (u_a = u_l = 0).

Natural frequency of the system with these parameters is equal to:

$$f_{12} = \frac{1}{2\pi}\sqrt{\frac{p(I_1 + I_2)}{I_1 I_2}} = 4.36\ Hz$$

Partial natural frequencies f_1 and f_2 characterizing the elastic vibrations of the system with fixed first and second masses are calculated according to the formulas:

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{p}{I_1}} = 3.56\,Hz,\; f_2 = \frac{1}{2\pi}\sqrt{\frac{p}{I_2}} = 2.52\,Hz.$$

Let us calculate parameters of two-loop control system [1]. In order to do that we will consider a rigid system with total moment of inertia $I_c = I_1 + I_2$.

Let us take a motor current loop time constant $T_{\mu T}$ = 0.005 s, and motor current loop damping coefficient a_T = 2. In this case, the maximum reachable motor velocity loop bandwidth will be equal to:

$$\omega_s = T_{\mu s}^{-1} = (a_T T_{\mu T})^{-1} = 100\,rad/s.$$

Let us take a motor velocity damping coefficient a_s = 2 and calculate motor speed loop gain with following formula:

$$b_c = \frac{I_c R \omega_s}{k_m k_y k_s a_s} = 0.6818.$$

Maximum reachable second mass angular position loop bandwidth will be equal to:

$$\omega_p = T_{\mu p}^{-1} = (a_s T_{\mu s})^{-1} = \frac{\omega_s}{a_s} = 50\,rad/s.$$

We can calculate second mass position loop gain taking second mass position loop damping coefficient a_p = 2 with following formula:

$$b_p = \frac{\omega_p k_c}{k_p a_p} = 25.$$

Step response of rigid system with this two-loop controller is presented in Fig. 1.

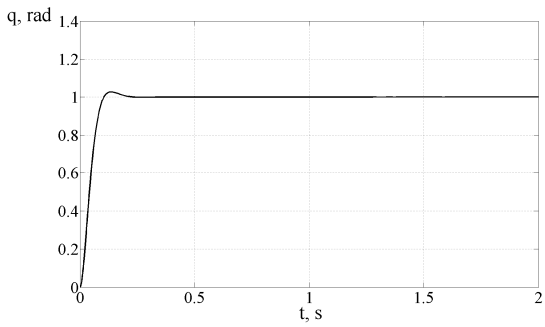

Fig. 1 Rigid system step response

Elastic system is unstable with this two-loop controller, therefore we need to reduce second mass position loop gain in 10 times. Step response of elastic system with this two-loop controller and reduced gain is presented in Fig. 2.

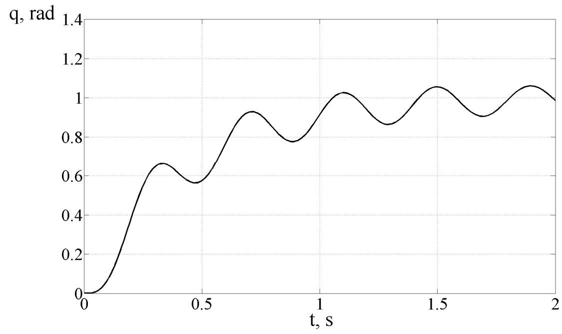

Fig. 2 Elastic system step response

3 FSF CONTROLLER

We can suppress elastic oscillations using FSF controller with following formula:

$$u_l = Kx,$$

where K = [K_1 K_2 K_3 K_4] – real numbers vector of FSF controller coefficients and x – state vector of the system (1). $x = [q_2\ \omega_2\ \tau_e\ \omega_1]^T$. Proportionality factor b in system with FSF controller can be found as b = $k_p - K_1$.

Let us take take a bandwidth of system with FSF controller equal to bandwidth of system with two-loop controller, i.e. 50 rad/s. And take desired characteristic polynomial of the system in the form of Newton's polynomial with following formula:

$$\varphi(\lambda) = \lambda^4 + 4\omega_0\lambda^3 + 6\omega_0^2\lambda^2 + 4\omega_0^3\lambda + \omega_0^4, \quad (2)$$

where ω_0 – desired system bandwidth.

Calculated vector K has following coefficients: K = [-5.6593 -0.4796 -0.1519 -0.0107]. Step response of elastic system is presented in Fig. 3.

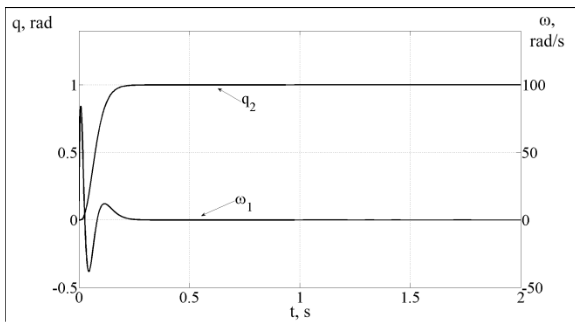

Fig. 3 Step response of elastic system with FSF controller

Step response of elastic system with decreased link stiffness in 5 times is presented in Fig. 4.

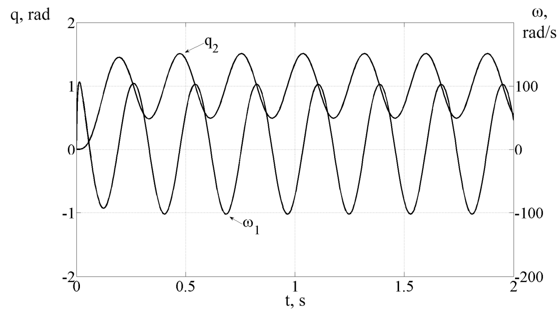

Fig. 4 Step response of elastic system with decreased link stiffness in 5 times

As can be seen from the Figures above, the FSF controller can suppress elastic oscillations with nominal parameters of the system, but parameter changes can lead to a serious deterioration in the quality of the system step response. In order to deal with parameter uncertainty we will add an adaptive control system with reference model to the system with FSF controller.

4 ADAPTIVE CONTROL SYSTEM

In order to improve system step response quality with parameters changing we will add an adaptive control system with reference model. Adaptive control action can be calculated by following formula:

$$u_a = K_A x + K_B u_0,$$

where K_A and K_B are tuning coefficients of adaptive controller. Differential equations of these coefficients are as follows [2][3]:

$$\begin{cases} \dot{K}_A(t) = -\Gamma_A B_M^T Pex^T - \Lambda_A K_A(t) \\ \dot{K}_B(t) = -\Gamma_B B_M^T Peu_0^T - \Lambda_B K_B(t) \end{cases}, \quad (3)$$

where $e = x - x_M$ – error vector that is equal to difference between system state vector and reference state vector; B_M – reference model input matrix; Γ_A and Γ_B – adaptive controller gains; Λ_A and Λ_B – adaptive controller feedback coefficients; P – symmetric positive definite matrix, that can be found from Lyapunov equation:

$$A_M^T P + P A_M = -G, \quad (4)$$

where G – arbitrary symmetric positive definite matrix. Let us take system (1) with FSF controller and nominal parameters as a reference model. In this case A_M and B_M will be as follows:

$$A_M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & -25 & 0 & 25 \\ -24973 & -1798.5 & -589.6902 & -199.95 \end{bmatrix},$$

$$B_M = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3750 \end{bmatrix}.$$

In most cases G is chosen to be a diagonal matrix with positive diagonal elements. Let us consider two adaptive control systems with following G matrices: G_1 = diag{10, 10, 10, 10,} and G_2 = diag{0.1, 1, 50, 0.001}. Let us find corresponding P matrices using Lyapunov equation (4):

$$P_1 = \begin{bmatrix} 1795.1 & 133.6168 & 31.6623 & 2.0022\cdot10^{-4} \\ 133.6168 & 11.7231 & 2.6549 & 0.0402 \\ 31.6623 & 2.6549 & 1.1595 & 0.0535 \\ 2.0022\cdot10^{-4} & 0.0402 & 0.0535 & 0.0317 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 1959.3 & 156.2915 & 7.819 & 2.0022\cdot10^{-6} \\ 156.2915 & 16.2812 & 1.1906 & 0.0706 \\ 7.819 & 1.1906 & 0.6569 & 0.0626 \\ 2.0022\cdot10^{-6} & 0.0706 & 0.0626 & 0.0078 \end{bmatrix}.$$

Let us take adaptive controller gains and adaptive controller feedback coefficients equal to 1 and simulate behavior of adaptive control systems with these two P matrices and different sets of system (1) parameters. Step responses of the adaptive systems with decreased link stiffness in 5 times are presented in Fig. 5. The dashed line represents reference model step response.

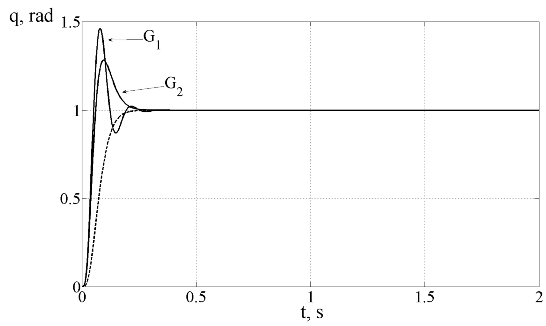

Fig.5 Step responses of the adaptive systems with decreased link stiffness in 5 times

Step responses of the adaptive systems with increased link stiffness in 5 times are presented in Fig. 6. The dashed line represents reference model step response.

Step responses of the adaptive systems with decreased second mass moment of inertia in 5 times are presented in Fig. 7. The dashed line represents reference model step response.

Step responses of the adaptive systems with increased second mass moment of inertia in 5 times are presented in Fig. 8. The dashed line represents reference model step response.

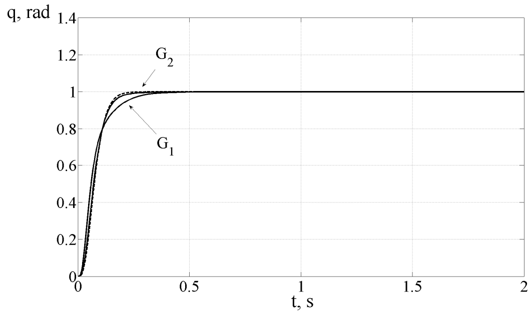

Fig. 6 Step responses of the adaptive systems with increased link stiffness in 5 times

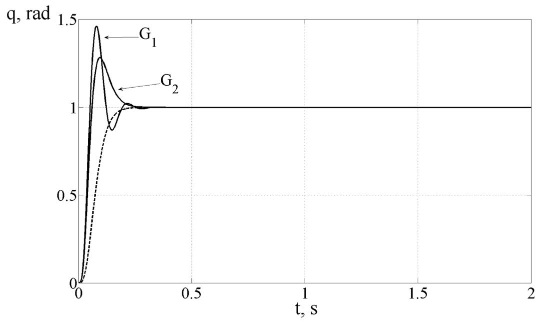

Fig. 7 Step responses of the adaptive systems with decreased second mass moment of inertia in 5 times

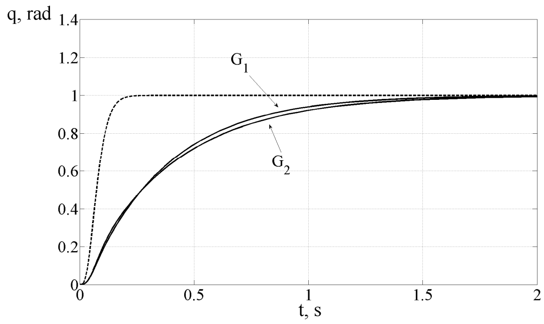

Fig. 8 Step responses of the adaptive systems with increased second mass moment of inertia in 5 times

As can be seen from the Figures above, system remains stable with parameter changing and using of adaptive controller. But step responses do not improve with increasing of adaptive controller gains and feedback coefficients and remain far from reference model step response. Lyapunov equation solution guarantees a stability of a control system, but it is difficult to find good elements of matrix G to get better step response of the system with different sets of system parameters.

5 GENETIC ALGORITHMS

Let us try to find matrix P using genetic algorithms [4]. Note that in equations (3), the matrix P is multiplied by a transpose reference model input matrix. In such systems reference model input matrix always has only one non-zero element, therefore only one row in matrix P affect adaptive control action. P is 4 by 4 matrix, thus we have to find only 4 coefficients of P matrix. Each individual of genetic algorithm population represents a set of 4 coefficients of P matrix. Thus, the individual chromosome contains four genes encoded in the form of real numbers. The initial population of individuals based on the matrix P coefficients calculated with the Lyapunov equation. Let us form minimized objective function of the genetic algorithm in the following manner:

1. Simulate step response of the system with fixed integration step and increased and decreased link stiffness and second mass moment of inertia in 3 times;
2. Calculate the sum of absolute values of differences between system step responses and reference model step responses of every integration point;
3. As a result of objective function for each individual take the largest calculated value in step 2 for each particular individual.

Using this objective function following coefficients of matrix P were obtained: $P_1 = 0$, $P_2 = 1.258$, $P_3 = 0.119$, $P_4 = 0.015$.

First three rows of P matrix are not involved in the control action calculation, thus they can be taken arbitrary. Step response of the adaptive system with genetic algorithm tuning and with decreased link stiffness in 5 times is presented in Fig. 9. The dashed line represents reference model step response.

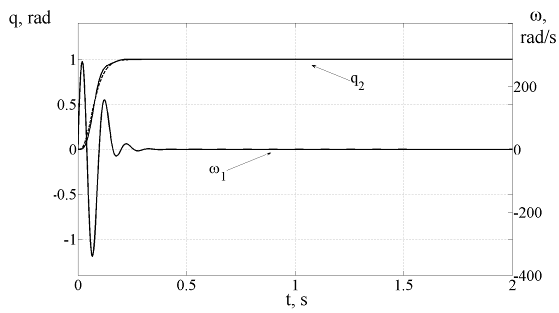

Fig. 9 Step response of the adaptive system with genetic algorithm tuning and with decreased link stiffness in 5 times

Step response of the adaptive system with genetic algorithm tuning and with increased link stiffness in 5 times is presented in Fig. 10. The dashed line represents reference model step response. Step response of the adaptive system with genetic algorithm tuning and with decreased second mass moment of inertia in 5 times is presented in Fig. 11. The dashed line represents reference model step response.

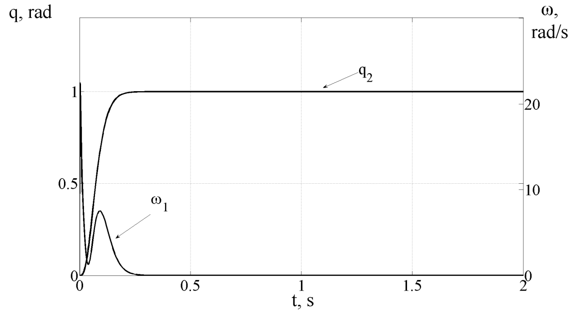

Fig. 10 Step response of the adaptive system with genetic algorithm tuning and with increased link stiffness in 5 times

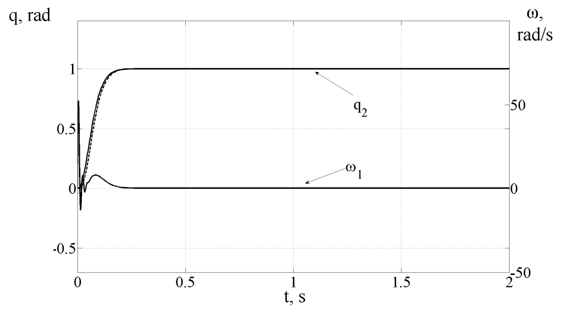

Fig. 11 Step response of the adaptive system with genetic algorithm tuning and with decreased second mass moment of inertia in 5 times

Step response of the adaptive system with genetic algorithm tuning and with increased second mass moment of inertia in 5 times is presented in Fig. 12. The dashed line represents reference model step response.

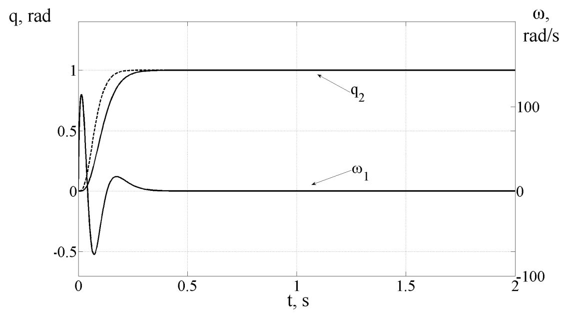

Fig. 12 Step response of the adaptive system with genetic algorithm tuning and with increased second mass moment of inertia in 5 times

As can be seen from the step responses, the use of genetic algorithms allows achieving better forms of step responses with system parameters changing.

6 STATE OBSERVER

Considered above adaptive and FSF controllers can be implemented only with direct measurement of all state variables. In most cases, second mass speed and elasticity torque are not available for direct measurement. Therefore an asymptotic state observer is required to implement these control algorithms. Let us construct state observer using motor speed with following formulas:

$$\dot{\hat{x}} = A\hat{x} + lc^T(\hat{x} - x) + bu,\ l = [l_1 l_2 l_3 l_4]^T, \quad (4)$$

where $\hat{x}$ – state estimations, l – observer feedback coefficients, c – system output matrix. In order to calculate these coefficients we will form characteristic polynomial for system (4):

$$\varphi(\lambda) = \det(A + lc^T - \lambda E),$$

where E – identity matrix. Let us take desirable characteristic polynomial in a form (2) and take desirable observer bandwidth in 3 times higher than FSF controller bandwidth. Calculated vector l has following values: $l =$

$$[-41.5462\ 4.1044 \cdot 10^3\ 1.4499 \cdot 10^4\ -439.8517]^T.$$

Step response of the system (1) with adaptive controller calculated using matrix $G = diag\{10, 10, 10, 10\}$, state observer and with decreased link stiffness in 4 times is presented in Fig. 13.

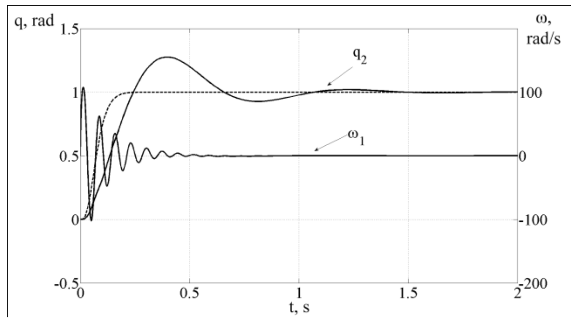

Fig. 13 Step response of the adaptive system with state observer and with decreased link stiffness in 4 times

Step response of the system (1) with adaptive controller calculated using matrix $G = diag\{10, 10, 10, 10\}$, state observer and with increased link stiffness in 4 times is presented in Fig. 14.

Step response of the system (1) with adaptive controller calculated using matrix $G = diag\{10, 10, 10, 10\}$, state observer and with decreased second mass moment of inertia in 4 times is presented in Fig. 15.

Step response of the system (1) with adaptive controller calculated using matrix $G = diag\{10, 10, 10, 10\}$, state observer and with increased second mass moment of inertia in 4 times is presented in Fig. 16.

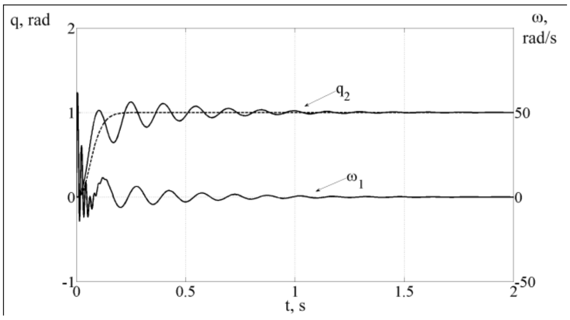

Fig. 14 Step response of the adaptive system with state observer and with increased link stiffness in 4 times

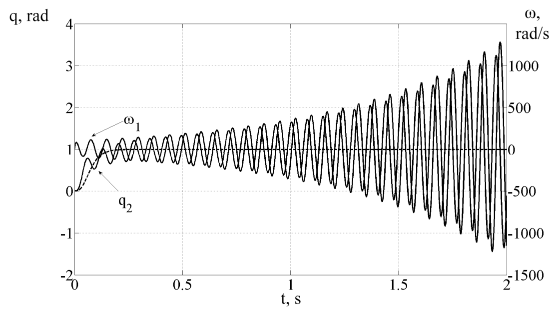

Fig. 15 Step response of the adaptive system with state observer and with decreased second mass moment of inertia in 4 times

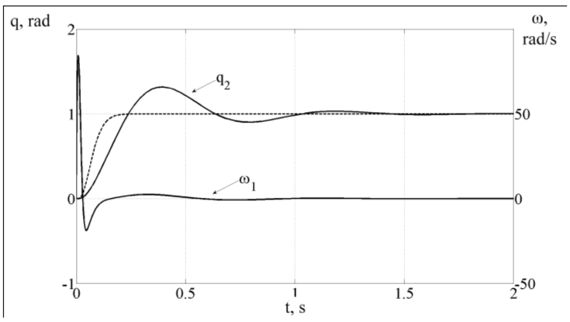

Fig. 16 Step response of the adaptive system with state observer and with increased second mass moment of inertia in 4 times

As can be seen from the figures above, parameters changing can make system unstable. We can obtain rapidly increasing oscillations using matrix $G = diag\{0.1, 1, 50, 0.001\}$ for adaptive controller calculation and changing system parameters the same way that we did above. Therefore matrix G can significantly affect behavior of adaptive control system with state observer. Let us try to find matrix P coefficients and l vector coefficients by genetic algorithm in the same time. We will use the same method that was used for matrix P coefficient searching, but this time each individual represents set of 8 parameters – matrix P coefficients and l vector coefficients. As a result we obtain the following coefficients: $l = [1170\ 46050 - 40 - 20230]^T$, $P_1 = 0$, $P_2 = 2.72$, $P_3 = 0.408$, $P_4 = 0.015$.

Step response of the system (1) with adaptive controller and state observer calculated using genetic algorithm and with decreased link stiffness in 4 times is presented in Fig. 17.

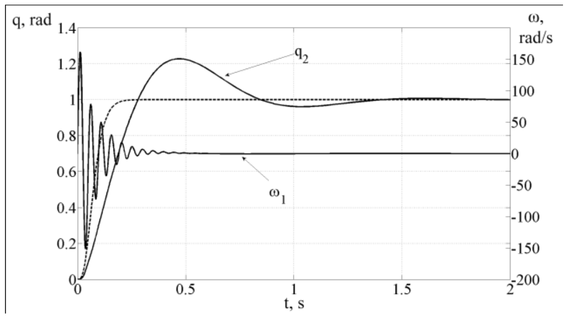

Fig. 17 Step response of the adaptive system and state observer calculated using genetic algorithm and with decreased link stiffness in 4 times

Step response of the system (1) with adaptive controller and state observer calculated using genetic algorithm and with increased link stiffness in 4 times is presented in Fig. 18.

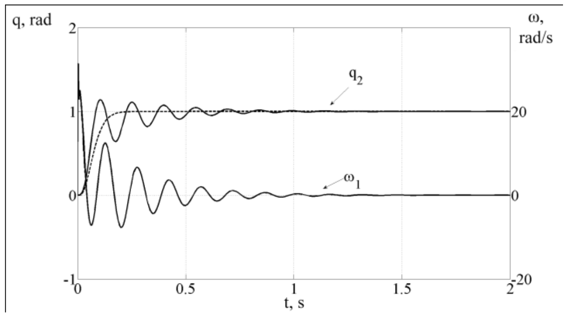

Fig. 18 Step response of the adaptive system and state observer calculated using genetic algorithm and with increased link stiffness in 4 times

Step response of the system (1) with adaptive controller and state observer calculated using genetic algorithm and with decreased second mass moment of inertia in 4 times is presented in Fig. 19.

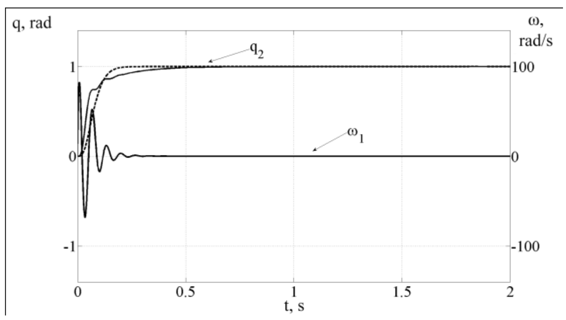

Fig. 19 Step response of the adaptive system and state observer calculated using genetic algorithm and with decreased second mass moment of inertia in 4 times

Step response of the system (1) with adaptive controller and state observer calculated using genetic algorithm and with increased second mass moment of inertia in 4 times is presented in Fig. 20.

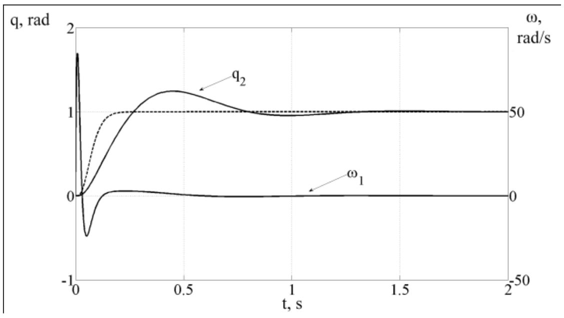

Fig. 20 Step response of the adaptive system and state observer calculated using genetic algorithm and with increased second mass moment of inertia in 4 times

The resulting system provides satisfactory step responses measuring only motor speed and second mass position with parameter changing.

7 CONCLUSION

Presented optimization techniques for simplified adaptive control system with parameter tuning, state observer and reference model using genetic algorithms can achieve a higher quality of system step responses.

8 REFERENCES

[1] Bortsov, Y. A.; Polyahov N. D.; Putov V. V.: “Electromechanical systems with adaptive and FSF control,” L.: Energoatomizdat, p. 216,, 1984.

[2] Putov, V. V.: “Adaptive control for dynamic systems: systems with reference models,” SPb.: Publisher Saint Petersburg Electrotechincal University “LETI”, p. 92, 2001.

[3] Putov, V. V.: “Methods for design of adaptive control systems for nonlinear time-variant dynamic plants with functional-parameter uncertainties,” Saint Petersburg Electrotechnical University “LETI”,p. 590, 1993.

[4] Ayman A. Aly: “PID Parameters Optimization Using Genetic Algorithm Technique for Electrohydraulic Servo Control System,” Intelligent Control and Automation, pp. 69-76, 2011.

Author:	Ignatev, Konstantin
University:	Saint Petersburg Electrotechnical University “LETI”
Department:	Control Systems
E-Mail:	kosign@yandex.ru